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Peristaltic Transport of Couple Stress Fluids in A Uniform and Non-Uniform Annulus through Porous Media

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Abstract: The Problem of peristaltic transport of a couple stress fluids through porous media in a gap between coaxial tubes where the outer tube is non-uniform with sinusoidal wave travelling down its wall and the inner tube is rigid. The relation between the pressure gradient, friction force on the inner and outer tube are obtained in terms of couple stress parameter. The numerical solution of pressure gradient, outer and inner friction force and flow rate are shown graphically.

Keywords: Peristaltic transport, couple stress parameter, porous media and flow rate.

INTRODUCATION

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a mechanism may be involved in swallowing food through the oesophagus, in urine transport form the kidney to the bladder through the urethra, in movement of chime in the gastro –intestinal tract, in the transport of spermatozoa in the ductus efferent of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels, and in the vasomotor of small blood vessel such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

A number of analytical in[1-7], numerical and experimental [8-12] studies of peristaltic flows of different fluids have been reported. A summary of most of the investigation reported up to the year 1983, has been presented by Srivastava and Srivastava[13] and some important contribution of recent year are referenced in Srivastava and Saxsen[14]. Physiological organs are generally observed have the form of a non-uniform duct studied by Ramachandra and Usha[5] and Wiedeman[12]. Hence, peristaltic analysis of a Newtonian fluid in a uniform geometry cannot be applied when explaining the mechanism of transport of fluid in most bio-systems. Recently Srivastava and Srivastava[15] and Srivastava and Srivastava[6] studied peristaltic transport of Newtonian and non-Newtonian fluids in non-uniform geometries. Rathod and Asha[16] studied the effects of magnetic field and an endoscope on peristaltic motion and Rathod and Laxmi Devindrappa[17] studied the slip effect on peristaltic transport of a porous medium in an asymmetric vertical channel by Adomian Decomposition method.

With the above discussion in mind, we propose to study the peristaltic transport of a viscous incompressible fluid (creeping flow) through the gap between coaxial tubes, where the outer tube is non-uniform and has a sinusoidal wave travelling down its wall and the inner one is a rigid, uniform tube and moving with a constant velocity. This investigation may have application in many clinical applications such as the endoscopes problem.

FORMULATION OF THE PROBLEM

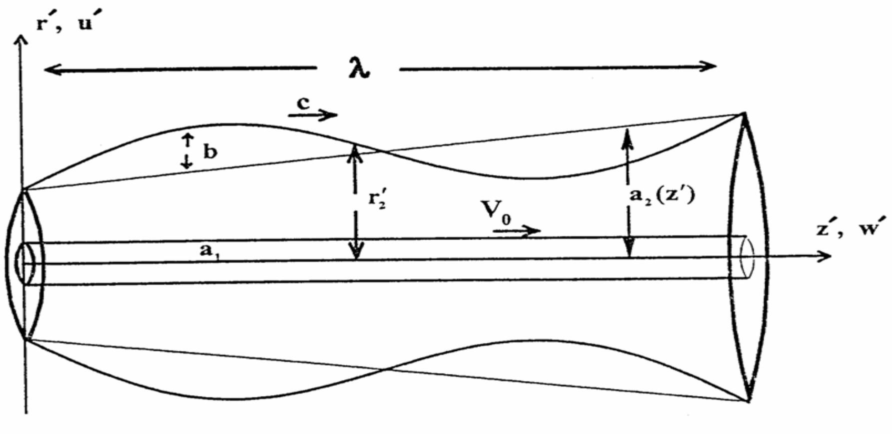
Consider the flow of an incompressible Newtonian fluid through coaxial tubes such that the outer tubes is non-uniform and has a sinusoidal wave traveling down and inner one rigid and moving with a constant velocity. The geometry of the wall surface is

$$r_1' = a_1 , \tag{1}$$

$$r_2' = a_2 + bSin(\frac{2\pi}{\lambda}(x' - ct)) \tag{2}$$

With $a_2(z') = a_{20}kz'$

With a_1 is the radius of the inner tube $a_2(z')$ is the radius of the outer tube at axial distance z' from inlet, a_{20} is the radius of the outer tube at the inlet, k (<<1) is the constant whose magnitude depends on the length of the outer tube, b is the amplitude, λ is the wave length, c is the propagation velocity, and t is the time. We choose a cylindrical coordinate system (r', z') where the z-axis lies along the centreline of the inner and the outer tubes and r' is the distance measured radially.



The equation of motion of the flow in the gap between the inner and the outer tubes are,

$$\frac{1}{r'}\frac{\partial(r',u')}{\partial r'}+\frac{\partial(w')}{\partial z'}=0 \tag{3}$$

$$\rho\{\frac{\partial u'}{\partial t'}+u'\frac{\partial u'}{\partial r'}+w'\frac{\partial u'}{\partial z'}\}=-\frac{\partial p'}{\partial r'}+\mu\{\frac{\partial}{\partial r'}(\frac{1}{r'}\frac{\partial(r',u')}{\partial r'})+\frac{\partial^2 u'}{\partial z'^2}\}-\eta\nabla^2(\nabla^2(u'))-\frac{\mu}{k}u' \tag{4}$$

$$\rho\{\frac{\partial w'}{\partial t'}+u'\frac{\partial w'}{\partial r'}+w'\frac{\partial w'}{\partial z'}\}=-\frac{\partial p'}{\partial z'}+\mu\{\frac{\partial}{\partial r'}(\frac{1}{r'}\frac{\partial(r',w')}{\partial r'})+\frac{\partial^2 w'}{\partial z'^2}\}-\eta\nabla^2(\nabla^2(w'))-\frac{\mu}{k}w' \tag{5}$$

Where $\nabla^2=\{\frac{\partial}{\partial r'}[\frac{1}{r'}(\frac{\partial(r')}{\partial r'})]\}$

Where u' and w' are the velocity components in the r' and w' direction respectively, ρ is the density, p' is the pressure and μ is the viscosity, η is the couple stress parameter and k is permeability of the porous media.

The boundary conditions are

$$u'=0, \qquad w'=V_0', \qquad \nabla^2(w')=0 \qquad at \qquad r'=r_1' \tag{6a}$$

$$u'=\frac{\partial r_2'}{\partial t'} \qquad w'=0 \qquad \nabla^2(w')=0 \qquad at \qquad r'=r_2' \tag{6b}$$

It is convenient to non-dimensionalize the variable appearing in equation (1-6) and introducing Reynolds number Re, wave number ratio δ and velocity parameter V_0 as follows:

$$z=\frac{z'}{\lambda'}, \quad r=\frac{r'}{c}, \quad u=\frac{\lambda u'}{a_{20}c}, \quad p=\frac{a_{20}^2}{\lambda\mu c}p'(z'), \quad t=\frac{t'c}{\lambda}, \quad \mathrm{Re}=\frac{\rho c a_{20}}{\mu},$$

$$\delta=\frac{a_{20}}{\lambda}, V_0=\frac{V'_o}{c} \quad r_1=\frac{r'_1}{a_{20}}=\varepsilon, \quad r_2=\frac{r'_2}{a_{20}}=1+\frac{\lambda kz}{a_{20}}+\phi(\frac{2\pi}{\lambda}(z-t)) \tag{7}$$

Where $\phi\,(\text{amplitude ratio})=\dfrac{b}{a_{20}}\le 1$

The equation of motion and boundary conditions in the dimensionless form becomes

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0 \tag{8}$$

$$\mathrm{Re}\,\delta^3\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\}=-\frac{\partial p}{\partial r}+\delta^2\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial(ru)}{\partial r})+\delta^4\frac{\partial^2 u}{\partial z^2}-$$
$$\frac{\delta^2}{\gamma^2}\nabla^2(\nabla^2(u))-\frac{\delta^2}{Da}u \tag{9}$$

$$\mathrm{Re}\,\delta\{\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial(rw)}{\partial r})+\delta^2\frac{\partial^2 w}{\partial z^2}-$$
$$\frac{1}{\gamma^2}\nabla^2(\nabla^2(w))-\frac{1}{Da}w \tag{10}$$

Where $\gamma=\sqrt{\dfrac{\eta}{\mu a_{20}^2}}$ is the couple-stress fluid parameter and $Da=\dfrac{k}{a_{20}^2}$ is a Darcy number using long wavelength $\delta \ll 1$ and negligible inertia (i.e., Re→0) approximations. The boundary conditions are;

$$u=0 \quad w=V_0 \quad \nabla^2(u,w)=0, \ \text{at} \quad r=r_1=\varepsilon, \tag{11a}$$

$$u=\frac{\partial r_2}{\partial y} \quad w=0 \quad \nabla^2(u,w)=0, \quad at\ r=r_2=1+\frac{\lambda kz}{a_{20}}+\phi\sin[2\pi(z-t)] \tag{11b}$$

Using the long wavelength approximation and dropping terms of order δ it follows from equation (8-11) that the appropriate equation describing the flow in the laboratory frame of reference are

$$\frac{\partial p}{\partial r}=0, \tag{12}$$

$$\frac{\partial p}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w}{\partial r})-\frac{1}{\gamma^2}\nabla^2(\nabla^2(w))-\frac{1}{Da}w \tag{13}$$

With dimensionless boundary condition

$w=V_0 \quad \nabla^2(u,w)=0 \qquad$ at $\; r=r_1=\varepsilon$,

$$w=0 \quad \nabla^2(u,w)=0 \qquad \text{at } r=r_2=1+\frac{\lambda kz}{a_{20}}+\phi\sin[2\pi(z-t)] \tag{14}$$

Integrating equation and using the boundary condition one finds the expression for the velocity profile as

$$\begin{aligned} w(z,t)=&-\frac{1}{4}(\frac{\partial p}{\partial z})\left\{(r_2^2-r_1^2)(\frac{In(r/r_1)}{In(r_2/r_1)})-r^2+r_1^2\right\}+\frac{1}{16\gamma^2}(\frac{\partial p}{\partial z})\left\{(r_2^2-r_1^2)^2(\frac{(In(r/r_1))^2}{(In(r_2/r))^2})-(r^2-r_1^2)^2\right\}\\ &+\frac{V_0}{In(r_2/r_1)}In(r/r_2)\left\{\frac{1}{4\gamma^2}\{(r_2^2-r_1^2)(\frac{In(r/r_1)}{In(r_2/r_1)})-r^2+r_1^2\}\right\}\\ &-2\frac{V_0}{In(r_2/r_1)}In(r/r_2)\left\{1-\frac{1}{Da}\right\} \end{aligned} \tag{15}$$

The instantaneous volume flow rate Q (z, t) is given by

$$\begin{aligned} Q(z,t)=\int_{r_1}^{r_2}2\pi rwdr=&-\frac{\pi}{8}\frac{\partial p}{\partial z}\left\{(r_2^2-r_1^2)\left[[r_2^2+r_1^2-(\frac{r_2^2-r_1^2}{In(r_2/r_1)})\right]\right\}-\pi V_O\left\{\frac{r_1^2-r_2^2}{In(r_2/r_1)}+r_1^2\right\}\left\{1-\frac{1}{Da}\right\}\\ &\frac{\pi}{32\gamma^2}\left\{(r_2^2-r_1^2)^2\left[(r_2^2+r_1^2)^2-\frac{(r_2^2-r_1^2)^2}{In(r_2/r_1)^2}\right]\right\}+\frac{\pi V_0}{8\gamma^2}\left\{\frac{r_1^2-r_2^2}{2In(r_2/r_1)}+r_1^2\right\}\\ &\left\{(r_2^2-r_1^2)^2\left[(r_2^2+r_1^2)^2-\frac{(r_2^2-r_1^2)^2}{In(r_2/r_1)}\right]\right\} \end{aligned} \tag{16}$$

Or

$$\frac{\partial p}{\partial z}=-8\left\{\frac{Q/\pi\;\; In(r_2/r_1)+\frac{V_0}{2}(r_1^2-r_2^2)\left[1-\frac{1}{Da}\right]+V_0r_1^2In(r_2/r_1)\left\{1+\frac{1}{8\gamma^2}\left[(r_2^4-r_1^4)\;-\frac{(r_2^2-r_1^2)^2}{In(r_2/r_1)}\right]\right\}}{(r_2^4-r_1^4)In(r_2/r_1)-(r_2^2-r_1^2)^2\left\{1+\frac{1}{4\gamma^2}\left[(r_2^4-r_1^4)\;-\frac{(r_2^2-r_1^2)^2}{In(r_2/r_1)}\right]\right\}}\right\} \tag{17}$$

The pressure rise $\Delta p_L(t)$ and friction force (at the wall) on the outer and the inner tubes $F_L^{(o)}(t)$ *and* $F_L^{(i)}(t)$ respectively, in a tube of length L, in their non-dimensional forms, are given by

$$\Delta p_L(t)=\int_0^A\frac{\partial p}{\partial z}dz \tag{18}$$

$$\Delta F_L^{(o)}(t)=\int_0^A r_2^2\left(-\frac{\partial p}{\partial z}\right)dz, \tag{19}$$

$$\Delta F_L^{(i)}(t)=\int_0^A r_1^2\left(-\frac{\partial p}{\partial z}\right)dz, \tag{20}$$

Where A=L/λ,

Substituting from equation (17) in equation (18-20) and with $r_1=\varepsilon$ and $r_2(z,t)=1+\frac{\lambda kz}{a_{20}}+\phi\sin[2\pi(z-t)]$, we get

$$\Delta p_L(t)=\int_0^A -8\{\frac{Q(z,t)}{\pi}In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]$$

$$+\frac{V_0}{2}[(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2-\varepsilon^2]\left[1-\frac{1}{Da}\right]+V_0\varepsilon^2 In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]\}$$

$$\{1+\frac{1}{8\gamma^2}X\}\ \{1/[(([1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t])^4-\varepsilon^4)$$

$$In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}](1+\frac{1}{4\gamma^2}\{(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2-\varepsilon^2)$$

$$[1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2+\varepsilon^2-(\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2-\varepsilon^2}{In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]})]\})\}dz \tag{21}$$

$$\Delta F_L^{(o)}(t)=\int_0^A\ 8\{(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2\}\ \{\frac{Q(z,t)}{\pi}In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]$$

$$+\frac{V_0}{2}[\varepsilon^2-(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2]\left[1-\frac{1}{Da}\right]+V_0\varepsilon^2 In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]\}$$

$$\{1+\frac{1}{8\gamma^2}X\}\ \{1/[(([1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t])^4-\varepsilon^4)$$

$$((1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2-\varepsilon^2)^2)[1+\frac{1}{4\gamma^2}$$
$$\{(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2-\varepsilon^2)[1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2+\varepsilon^2$$
$$-(\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2-\varepsilon^2}{In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]})]\}]\}dz\}\} \tag{22}$$

$$\Delta F_L^{(i)}(t)=\int_0^A 8\varepsilon^2\{\frac{Q(z,t)}{\pi}In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]$$
$$+\frac{V_0}{2}[\varepsilon^2-(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2]\left[1-\frac{1}{Da}\right]+V_0\varepsilon^2 In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]$$
$$\{1+\frac{1}{8\gamma^2}X\}\}\{1/[((1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^4-\varepsilon^4)$$
$$In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]\;-((1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2-\varepsilon^2)^2]$$
$$\{1+\frac{1}{4\gamma^2}\{(1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2-\varepsilon^2)[1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2+\varepsilon^2$$
$$-(\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)^2-\varepsilon^2}{In[\frac{1+\frac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t)}{\varepsilon}]})]\}\}\}dz \tag{23}$$

The limiting of equation (15-17) as r_1 tends to zero gives the forms of the axial velocity and the pressure gradient for peristaltic flow in non uniform tube(without endoscope, $\varepsilon=0$), these are

$$w(r,z,t)=-\frac{1}{4}(\frac{\partial p}{\partial z})(r_2^2-r^2)+\frac{1}{16}(\frac{\partial p}{\partial z})(r_2^4-r^4), \tag{24}$$

$$\frac{\partial p}{\partial z}=-\frac{8Q}{\pi r_2^4}+\frac{16Q}{\pi r_2^{16}}, \tag{25}$$

Hence the pressure rise and the outer friction force, in this case respectively, take the form

$$\Delta p_L(t) = -8\int_0^A \frac{[Q(z,t)/\pi]dz}{(1+\dfrac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^4} + 16\int_0^A \frac{[Q(z,t)/\pi]dz}{\gamma^2(1+\dfrac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^{16}} \quad (26)$$

$$\Delta F_L^{(o)}(t) = 8\int_0^A \frac{[Q(z,t)/\pi]dz}{(1+\dfrac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^2} + 16\int_0^A \frac{[Q(z,t)/\pi]dz}{\gamma^2(1+\dfrac{\lambda kz}{a_{20}}+\phi\sin 2\pi(z-t))^4} \quad (27)$$

If k=0 in equations (26) and (27), we get expression for the pressure rise and friction force in a uniform tube. The analytical interpretation of our analysis with other theories are difficult to make at this stage, as the integrals in equation (21-23) and equation (26) and (27) are not integrable in closed form, neither for non-uniform nor uniform geometry (k=0).Thus further studies of our analysis are only possible after numerical evaluation of these integrals.

RESULT, DISCUSSION AND CONCLUSION

To discuss the results obtained above quantitatively assume the form of the instantaneous volume rate of the flow Q (z, t), periodic in (z-t) as (14-16)

$$\frac{Q(z,t)}{\pi} = \frac{\bar{Q}}{\pi} - \frac{\phi^2}{2} + 2\phi\sin(2\pi(z-t)) + \frac{2\lambda kz}{a_{20}}\phi\sin(2\pi(z-t)) + \phi^2\sin^2(2\pi(z-t))$$

Where $\bar{Q}$ the time average of the flow is over one period of the wave .This form Q (z, t) has been assumed in view of the fact that the constant value of Q (z, t) gives $\Delta P_L(t)$ always negative, and hence will be no pumping action. Using this form of Q (z, t), we shall now compute the dimensionless pressure rise $\Delta P_L(t)$, the inner friction force $F_L^{(i)}(t)$ (on the inner surface) and the outer friction force $F_L^{(o)}(t)$ (on the outer tube) over the tube length for various value of the dimensionless time t, dimensionless flow average $\bar{Q}$, amplitude ratio ϕ, radius ratio ε, couple stress parameter γ and the velocity of the inner tube V_0.The average rise in pressure $\Delta \bar{P}_L$, outer friction force $F_{(L)}^{-(o)}$ and the inner friction force $F_L^{-(i)}(t)$ are then evaluated by averaging $\Delta P_L(t)$, $F_L^{(o)}(t)$ and $F_L^{(i)}(t)$ over one period of the wave. As integrals in equation (21-23) are not integrable in closed form, they are evaluated numerically using digital computer. Following us use the value of the various parameters in equation (21-23) as:

a_{20}=1.25cm, L= λ =8.01cm $k = \dfrac{3a_{20}}{\lambda}$.

Furthermore, since most routine upper gastrointestinal endoscopes are between 8-11 mm in diameter as reported Cotton and Williams [2] and the radius of the small intestine is 1.25 cm as reported in Srivastava and Saxena [16].

In **Fig (1) and (2)** represent the variation of dimensionless pressure rises with dimensionless time t for $\phi = 0.4, \varepsilon = 0.32, V_0 = 1$ and Da=1, for different values of γ for non-uniform and uniform annulus respectively. Figures show that as the couple stress fluid parameter γ increases the difference in pressure rise decreases for large values of γ.

Figure (3) and (4) illustrate the variation of dimensionless pressure rises with dimensionless time t for $\phi = 0.4, \varepsilon = 0.32, V_0 = 1$and $\gamma = 0.2$ for different values of Darcy number Da for uniform and non-uniform annulus. Figure shows that as the Darcy number increase there is decrease in the pressure rise. It is also notice that from figure that the maximum pressure rise $\Delta P_L(t)$ is obtained by putting dimensionless flow average $\bar{Q}$ =0.

Fig (5) - (8) shows the inner friction force (on the inner surface) and outer friction forces (on the outer surface) are plotted versus dimensionless time t for different values of γ and Da. We noticed that the inner and outer friction force is opposite behaviour compared with the pressure rise. It is also noticed that the inner friction force behaves similar to the outer friction force for the same values of the parameters. Moreover, the outer friction force is greater than the inner friction force at the same values of the parameters.

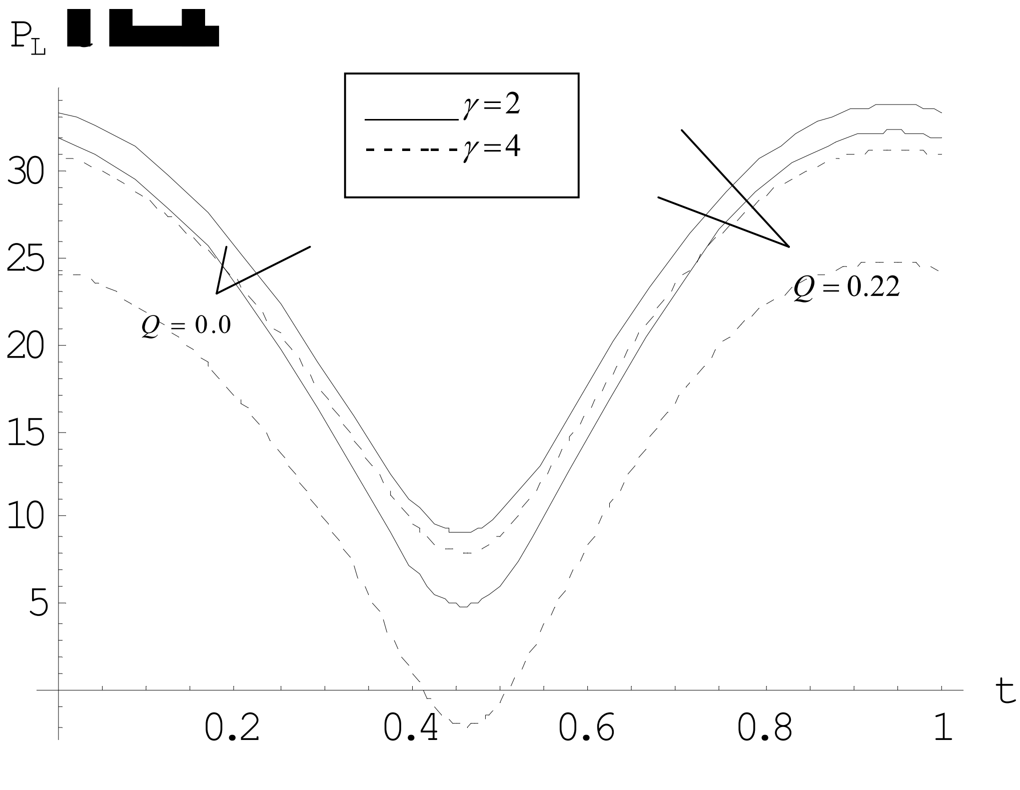


Fig. 1: Variation of of pressure rise over the length of a uniform annulus at $\varepsilon = 0.32$ $\phi = 0.4$, $V_0 = 1$ and Da=1 for different values of γ

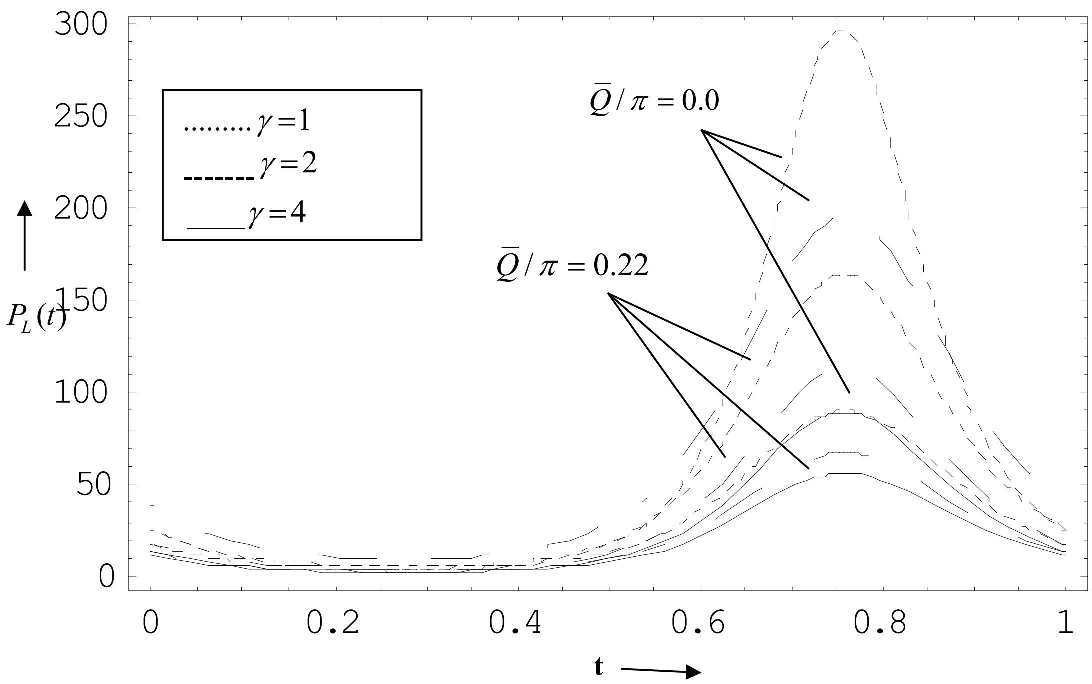


Fig. 2: Variation of pressure rise over the length of a non–uniform annulus at $\varepsilon=0.32$ $\phi=0.4$, $V_0=1$ and Da=1 for different values of γ

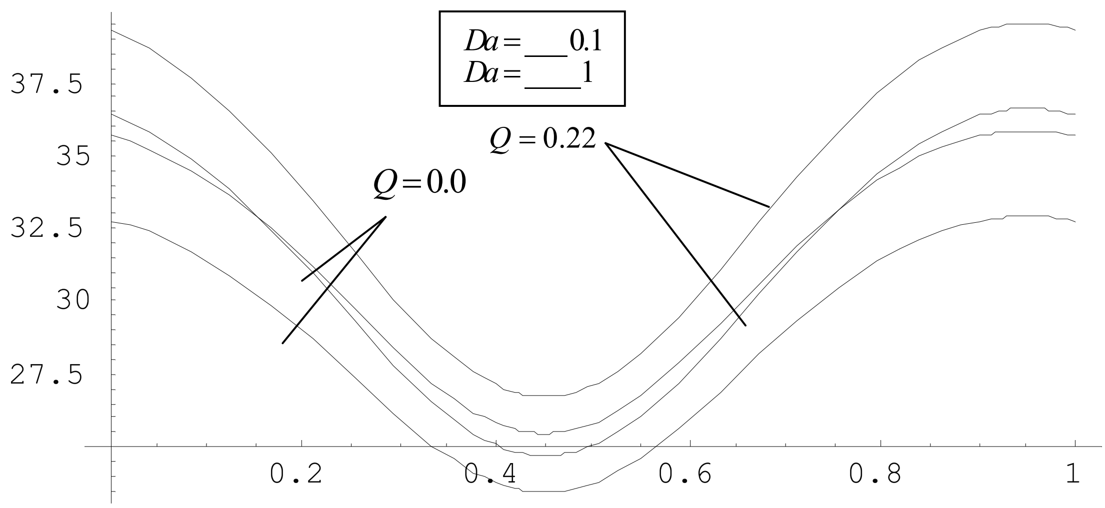


Fig. 3: Variation of pressure rise over the length of a uniform annulus at $\gamma=0.2$, $\phi=0.4$, $\varepsilon=0.32$ for different values of Da

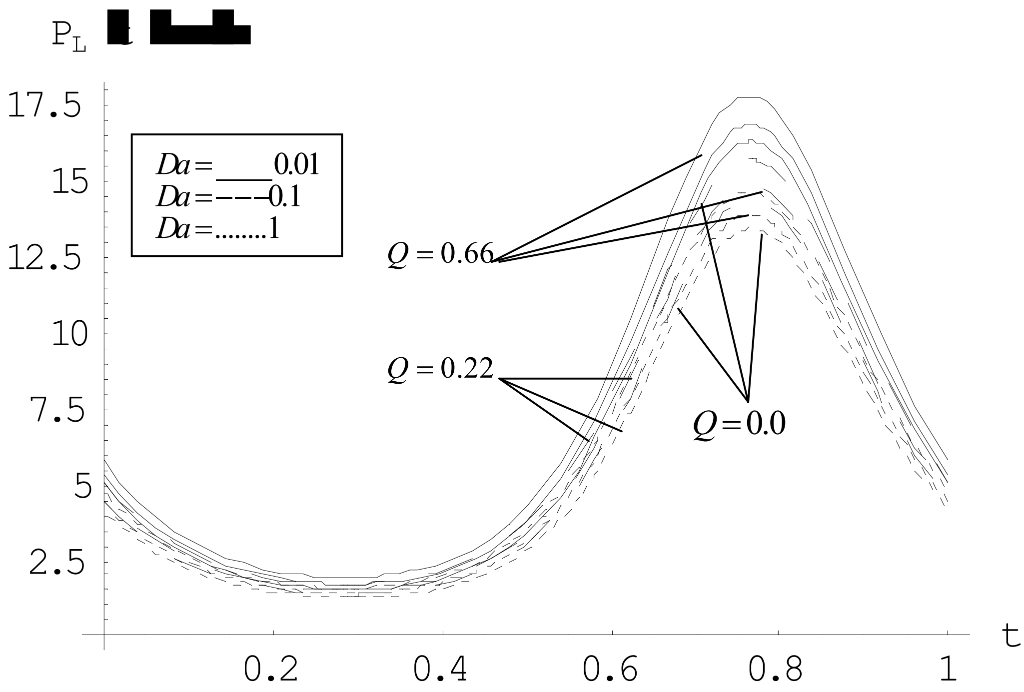
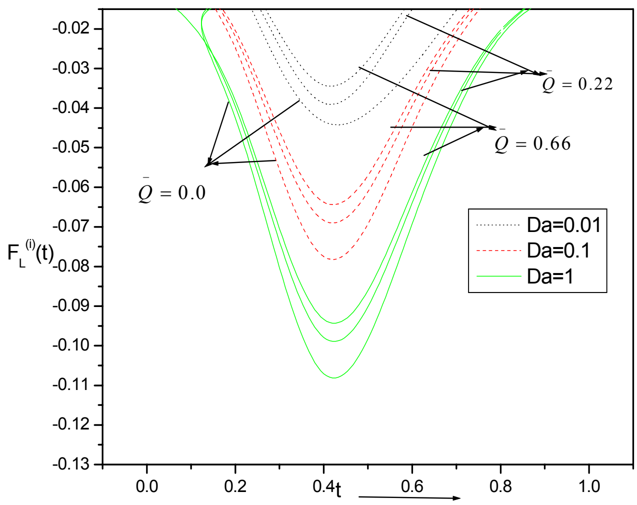
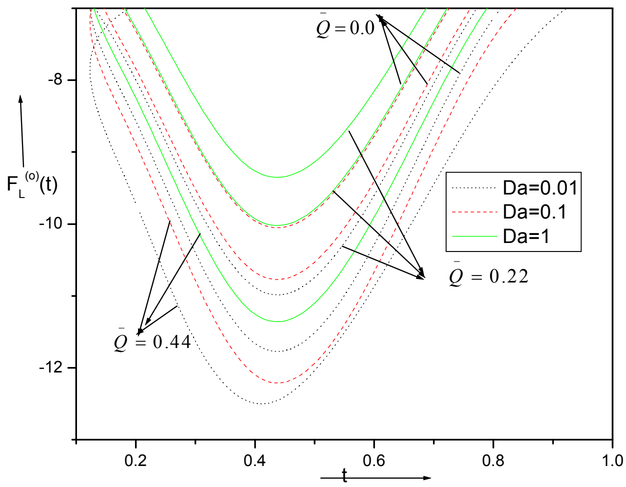


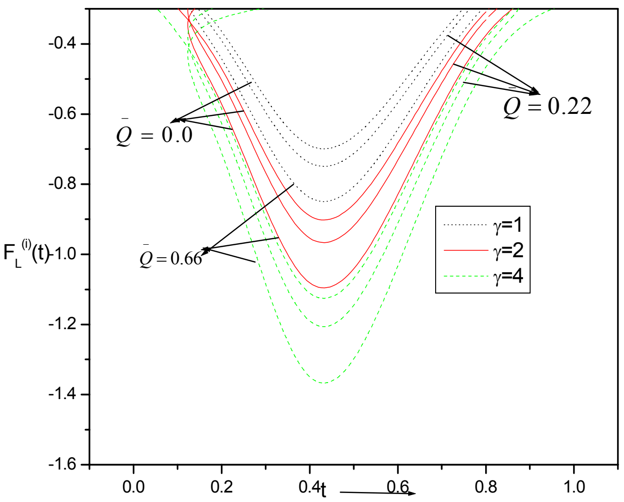
Fig. 4: Variation of pressure rise over the length of non uniform at at $\gamma = 0.2$ $\phi = 0.4$, $V_0 = 1$ and $\varepsilon = 0.32$ for different values of Da



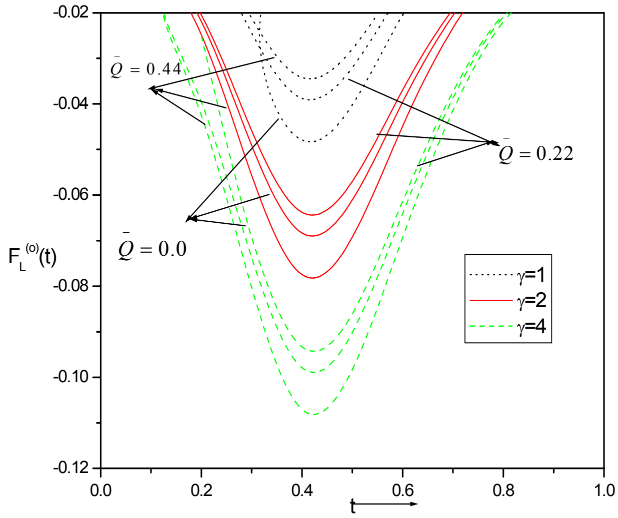
fig(5)Variation of inner friction force of non uniform annulus at $\gamma=0.2,\phi=0.4$ and $V_0=1$,for different values of Da at $\varepsilon=0.38$



fig(6)Variation of the outer force over the length of a non-uniform at $\gamma=0.2,\phi=0.4$, and $\varepsilon=0.38$ for different values of Da at $V_0=1$



fig(7) Variation of inner friction force over the length of non uniform annulus at Da=0.2, $\phi=0.4$, $V_0=1$ and $\varepsilon=0.32$for different values γ



fig(8)Variation of the outer friction force over the length of non-uniform annulus at $\gamma=0.2$, $\phi=0.4$ and $V_0=1$ for different values of γ at $\varepsilon=0.38$

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