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Research Article

Numerical Rate of Convergence of Bisection Method

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ABSTRACT

A computer program in C++ language has been developed to calculate cube roots of numbers from 1 to 25 in interval [1, 3] using bisection method. Rate of convergence has been found out in each calculation. Lowest rate of convergence has been observed in the evaluation of cube root of 18 and highest in the evaluation of cube root of 9. Average rate of convergence of bisection method has been found to be 2.488206405.

Key words: Bisection method, rate of convergence, Intermediate-Value theorem, algorithm.

INTRODUCTION

Numerical analysis involves the study of methods of computing numerical data. In many problems this implies producing a sequence of approximations; thus the questions involve the rate of convergence, the accuracy (or even validity) of the answer, and the completeness of the response. (With many problems it is difficult to decide from a program's termination whether other solutions exist.) Since many problems across mathematics can be reduced to linear algebra, this too is studied numerically; here there are significant problems with the amount of time necessary to process the initial data. Numerical solutions to differential equations require the determination not of a few numbers but of an entire function; in particular, convergence must be judged by some global criterion[1-5]. Other topics include numerical simulation, optimization, and graphical analysis, and the development of robust working code.

The Bisection Method calls for a repeated halving of subintervals of [a, b] and at each step locating the half containing the root. This procedure is best to use when we only have an interval in which the root is contained [6-8]. It will also work when there is more than one root in the interval; however, for this problem we assume the root is unique. This method's major drawback is that it's the slowest of the three methods to converge. However, the method always converges to a solution and would be good to use as a starter for one of the other methods.

MATERIAL AND METHOD

The bisection method is developed with the support of the *Intermediate-Value Theorem*. Let us consider a continuous function $f(x)$ over the interval $[a, b]$. If

$$f(a)f(b) < 0$$

Then the values of $f(a)$ and $f(b)$ must be nonzero and each of different sign. That is, if $f(a)$ is negative, $f(b)$ must be positive[9-13]. Likewise, if $f(a)$ is positive, $f(b)$ must be negative. In light of the Intermediate-Value theorem, the

Intermediate-value that we seek is zero, which certainly lies between the positive and negative numbers represented by $f(a)$ and $f(b)$. The conclusion of the Intermediate-Value theorem is that there *is* a point, ξ, between a and b where

$F(\xi) = 0$. That is, the solution to $f(x) = 0$ lies somewhere between a and b.

The bisection method checks the midpoint, c, of the interval $[a, b]$. If $f(c) = 0$, the task of finding a root is complete. If $f(c) \neq 0$, then for the same reason, there must be a root in either $[a, c]$ if $f(a) f(c) < 0$ or on $[c, b]$ if $f(c)f(b) < 0$.

This procedure defines the bisection method. At each step the interval in which there is guaranteed to be a root of the equation is halved (bisected), and the method terminates as soon as the width of the interval containing the root is less than some error tolerance $\delta > 0$. Since the bisection method keeps a bounded interval where there is at least one root at each step, it falls in the category of bracketing methods [14].

An algorithmic definition of the bisection method is as follows [7].

Inputs: the function *f(x)*, the initial interval [a, b] and the stopping tolerance δ.

```
If ((f (a) * f (b) ≥ 0) then
Begin
        c= (a + b) / 2
        While (f(c) < δ)
        Begin
                c= (a + b)/2
                If (sign f (a)) = (sign f(c)) then a=c else b=c
        End
        Return c;
End
Else
    Write (“Root does not exist”)
```

Computer program for bisection method developed by us is given below-

```
#include<conio.h>
#include<stdio.h>
#include<math.h>
//Bisection method
Void main (void)
{
  FILE *fpt;
```

```
Int n;
float a[1000],b[1000],c[1000],delta, rl, ru, d, aa;
Double f (float x);
Clrscr ();
//Filename to store result
fpt=fopen("shalb25.txt", "w");
rl=1; ru=3; n=1; a[1]=rl; b[1]=ru; aa=fabs(rl-ru);
  //to check existence of root between the interval
d=f (rl)*f(ru);
Delta=0.00001;
Fprintf (fpt,"    n       a[n]            b[n]          c[n]        f(c[n])\n");
Printf ("   n       a[n]            b[n]          c[n]        f(c[n])\n");
If (d<0)
{
  While (aa > delta)
 {
    If (a[n] == b[n]) break;
    c[n] = (a[n] +b[n])/2;
    If (((f(c[n])>0) && (f (a[n])>0)) || ((f(c[n])<0) && (f(a[n])<0)))
   {
          A [n+1]=c[n];
          b [n+1]=b[n];
    }
    Else
   {
          b [n+1]=c[n];
          a [n+1]=a[n];
    }
   Aa = fabs(f(c[n]));
   fprintf(fpt,"%3d %15.12f %15.12f %15.12f %18.12f\n", n,a[n],b[n],c[n],f(c[n]));
   printf("%3d %15.12f %15.12f %15.12f %18.12f\n",n,a[n],b[n],c[n],f(c[n]));
   If (aa > delta)   n=n+1;
 }
       Print f ("Root= %20.15f\n",c[n]);
       Print f ("Value of function=%20.15f\n", f(c[n]));
       Print f ("No. of iterations=%3d\n",n);
       Print f ("\n");
       Getch ();
}
Else
 {
Printf ("There is no root in the given interval\n");
```

```
      Getch ();
    }
  Fclose (fpt);
 }
//Function definition
Double f (float x)
{
  Double r;
  r=x*x*x-26;
  Return (r);
}
```

With the help of above computer program, cube roots of the number from 1 to 25 in the interval [1, 3] have been calculated. For this, the following functions have been taken

$$f(x) = x^3 - n \quad \text{where n = 1,2, 3, ……. , 25}$$

Numerical rate of convergence have been defined by us as follows–

$R_{bc} = 10^8$ / {(Number of iterations) (Value of function at the x = r i.e. $f(r)$)}

Where R_{bc} is the numerical rate of convergence of bisection method

and r is the root of function f(x) = 0 as estimated by secant method.

If $f(r) =0$ then R_{bc} has been assumed to 100.

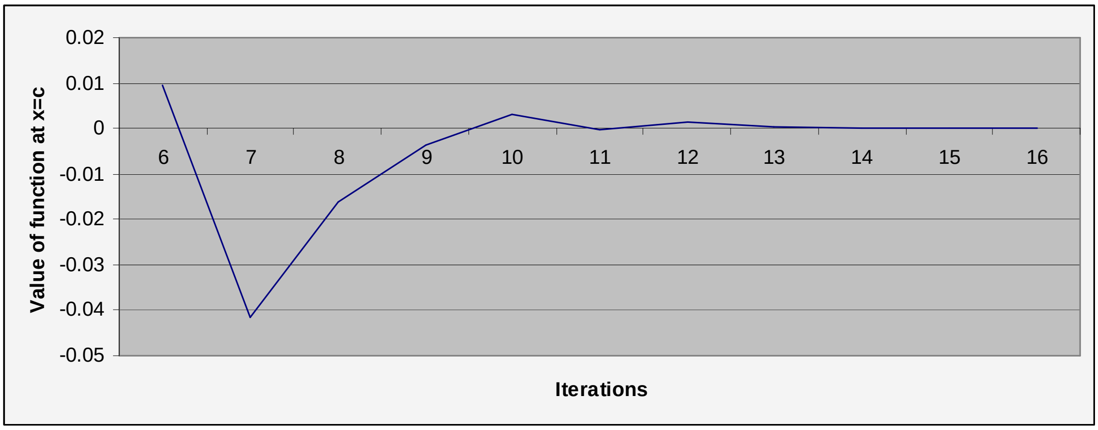
RESULT AND DISCUSSION

Cube roots of natural numbers from 1 to 25 have been calculated using bisection method in the interval [1, 3] with stopping tolerance 0.00001. For the calculation of cube root of 1, the interval [0.9, 3.1] has been taken as in this case the product $f(a) f(b)>0$. For the calculation of cube root of 8, the interval [1, 3.1] has been taken as in this case $f(2)=0$ and hence we have to assume R_{bc}=100. This will affect the average rate of convergence.

Bisection method has been applied to calculate the roots of equation.

$$f(x) = x^3 - 1 = 0$$

In the interval [0.9, 3] using computer program developed by us. Initial value, last value, middle point of interval and value of function at middle point of the interval in each iteration is included in **Table-1**. Graph showing the value of function at middle point x=c from iteration 6 to 16 is given in **Graph-1**.



Graph-1: Values of function at middle point x=c form iteration 6 to 16

Cube root of 1 as estimated by bisection method = 1.000003099442

Rate of convergence of bisection method in the calculation of cube root of 1

= 0.6721620485

In the similar manner, the cube roots of numbers from 2 to 25 have been calculated with the help of computer program by changing the definition of the function in computer program.

CONCLUSION

Lowest rate of convergence of bisection method has been observed in the calculation of cube root of 18 and is equal to 0.513604787. Highest rate of convergence of bisection method has been observed in the calculation of cube root of 9 and is equal to 16.05404433. Average rate of convergence of bisection method is 2.488206405**.** Rates of convergence and value of roots of the functions are shown in Table-2.

Table-1: Initial value, last value, middle point of interval and value of Function at middle point of the interval.

Number of iterations	Initial value (a)	Last value (b)	Middle point (c)	Value of function *f(x)* at x=c
1	0.899999976158	3.099999904633	2.000000000000	7.000000000000
2	0.899999976158	2.000000000000	1.450000047684	2.048625300765
3	0.899999976158	1.450000047684	1.174999952316	0.622234177500
4	0.899999976158	1.174999952316	1.037499904633	0.116771176413
5	0.899999976158	1.037499904633	0.968749940395	-0.090850997891
6	0.968749940395	1.037499904633	1.003124952316	0.009404183446
7	0.968749940395	1.003124952316	0.985937476158	-0.041597088724
8	0.985937476158	1.003124952316	0.994531214237	-0.016316797994
9	0.994531214237	1.003124952316	0.998828053474	-0.003511720810
10	0.998828053474	1.003124952316	1.000976562500	0.002932549454
11	0.998828053474	1.000976562500	0.999902307987	-0.000293047408
12	0.999902307987	1.000976562500	1.000439405441	0.001318795640
13	0.999902307987	1.000439405441	1.000170826912	0.000512568286
14	0.999902307987	1.000170826912	1.000036597252	0.000109795774
15	0.999902307987	1.000036597252	0.999969482422	-0.000091549940
16	0.999969482422	1.000036597252	1.000003099442	0.000009298353

Table-2: Rate of convergence of bisection method in the evaluation roots of functions $f(x) = x^3 - n = 0$; n=1, 2, …., 25

S. No.	Function	No. of Iterations	Value of root	Rate of convergence
1	$f(x)=x^3-1$	16	1.000003099	0.672162049
2	$f(x)=x^3-2$	19	1.259922028	1.130409000
3	$f(x)=x^3-3$	19	1.442249298	3.098401098
4	$f(x)=x^3-4$	12	1.587402344	4.905801738
5	$f(x)=x^3-5$	17	1.709976196	3.462918874
6	$f(x)=x^3-6$	19	1.817119598	0.534290882
7	$f(x)=x^3-7$	20	1.912931442	1.755222313
8	$f(x)=x^3-8$	19	2.000000238	1.839606985
9	$f(x)=x^3-9$	20	2.080083847	16.054044330
10	$f(x)=x^3-10$	17	2.154434204	0.869339151
11	$f(x)=x^3-11$	19	2.223979950	2.522446558
12	$f(x)=x^3-12$	14	2.289428711	2.011468427
13	$f(x)=x^3-13$	21	2.351334572	2.477496712
14	$f(x)=x^3-14$	21	2.410141945	0.855832748
15	$f(x)=x^3-15$	21	2.466212273	1.315970420
16	$f(x)=x^3-16$	20	2.519842148	5.464152428
17	$f(x)=x^3-17$	18	2.571281433	1.777792000
18	$f(x)=x^3-18$	21	2.620741844	0.513604787
19	$f(x)=x^3-19$	20	2.668401718	3.371910234
20	$f(x)=x^3-20$	21	2.714417458	1.354783292
21	$f(x)=x^3-21$	20	2.758924484	0.711212261
22	$f(x)=x^3-22$	21	2.802039146	1.097351638
23	$f(x)=x^3-23$	21	2.843867302	0.609346680
24	$f(x)=x^3-24$	22	2.884499073	2.694367495
25	$f(x)=x^3-25$	21	2.924017906	1.105228019
Average rate of convergence of bisection method				2.488206405

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