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Evaluation of the Binding and Excitation Energies in the Nuclear S-D Shell Model Theory

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Abstract: We use a theoretical shell-model calculation with a two-body effective interaction derived microscopically from the Wildenthal ,and new universal model-independent interactions in the SD (0s1d) shell USDA and USDB) potential to calculate the binding and excitation energies for ${}^{30}_{14}\mathrm{Si}_{16}$ at Low –Lying. Depending on this model, we calculated the energies by assume ${}^{28}_{14}\mathrm{Si}_{14}$ as inert core with two nucleon out of the core in the $2S_{1/2}$ $1d_{3/2}$ configuration. our results suggest the isospin T=1and T=0 for antisymmetric two particale wave functions .The effective two-body matrix elements are obtained from the Wildenthal ,USDA,and USDB interaction.A Matlab program has been used to employed the calculation of the energies with experimental single particle energies. Our results of the theoretical calculation show a good agreement with the experimental data.

Key words: Nuclear S-D shell,Binding and Excitation Energies, shell model theory

INTRODUCTION

The field of nuclear physics has grown rapidly since the discovery of the atomic nucleus in the early 1900.Each nucleus is comprised of number of protons(Z) and neutrons(N).these nucleons are quantum mechanical particles and thus the nucleus itself is a quantum mechanical many body system Church[1]. The nuclear shell model is based on the analogous model for the orbital structure of atomic electrons in atoms. In some areas it gives more detailed predictions than the other model. In principle, the shell model's energy level structure can be used to predict nuclear excited state Martin[2].

The shell model works only very well for nuclei with a magic number of protons and neutrons or a “valence" configuration with only one particle outside the core. But the shell structure of the majority of nuclei in the nuclide chart differs considerable from the theoretical predictions of the shell model

Neidherr[3]. The concept of symmetry in physics is a very powerful tool for the understanding of the behavior of nature. Symmetries are intimately related to conservation laws and to conserved quantities which, in quantum mechanics, translate into good quantum numbers.

In nuclear physics, several symmetries have been identified. In particular, isospin quantum number t, with symmetry is related to the identical behavior of protons $tz = -1/2$ and neutrons $tz = 1/2$ in the nuclear field Lenzi and Bentley [4]. In the study of nuclear structure properties, nuclear masses or binding energies (BE) and, more in particular, two-neutron separation energies ($S2$n), are interesting probes to find out about specific nuclear structure correlations that are present in the nuclear ground state Fossion *etal*[5]. In most cases non-relativistic kinematics is used. The bare nucleon-nucleon (or nucleon-nucleon-nucleon) interactions are inspired by meson exchange theories or more recently by chiral perturbation theory, and must reproduce the nucleon-nucleon phase shifts, and the properties of the deuteron and other few body systems. The challenge is to find Alfredo[6]. Maria G. Mayer's discussion of the magic numbers in nuclei has clearly demonstrated the nuclear shell structure associated with the independent-particle model for nuclei. In this model, each closed-shell. The basic assumption of the nuclear shell model is that to a first approximation each nucleon moves independently in a potential that represents the average interaction with the other nucleons in a nucleus Vesselin[7]. This independent motion can be understood qualitatively from a combination of the weakness of the long-range nuclear attraction and the Pauli Exclusion Principle. The complete Schrödinger equation for A nucleons reads Heyde[8].

$$\widehat{\mathrm{H}}\psi(1,2,3,\dots\dots,\mathrm{A}) = \mathrm{E}\psi(1,2,3,\dots\dots,\mathrm{A}) \qquad \dots (1)$$

Where $\widehat{\mathrm{H}}$ is the non relativistic Hamiltonian operator contains single nucleon kinetic energies and two-body interactions Kris[9].

$$\widehat{\mathrm{H}} = \sum_{i=1}^{A}\left[\frac{-\hbar^2}{2m}\nabla_i{}^2 + U(i)\right] + \sum_{i<j=1}^{A} W(i,j) - \sum_{i=1}^{A} U(i) = \widehat{H}^{(o)} + \widehat{H}_{res}^{(1)} \qquad \dots (2)$$

Where $\widehat{H}^{(o)}$ is the Hamiltonian of one body potential, and $\widehat{H}_{res}^{(1)}$ is the residual interaction.

Inserting of Eq. (2) in Eq. (1) results.

$$\left(\widehat{H}^{(o)} + \widehat{H}_{res}^{(1)}\right)\psi(1,2..\mathrm{A}) = \widehat{H}^{(o)}\psi(1,2,..\mathrm{A}) + \widehat{H}_{res}^{(1)}\psi(1,2,3,\dots\mathrm{A}) \qquad \dots (3)$$

Under the first order perturbation theory the wave function have been made in

$$\psi(1,2\dots,\mathrm{A}) = |\psi^0(1,2\dots,\mathrm{A})\rangle + |\psi^{res}(1,2\dots,\mathrm{A})\rangle \qquad \dots (4)$$

Results to energies.

$$E = E^0 + \mathrm{E}^{res} \qquad \dots (5)$$

Substituting Eq.(4) in Eq.(3) give zeroth and first order quantity one obtain Trun[10].

$$\left(\widehat{H}^{(o)} + \widehat{H}_{res}^{(1)}\right)\left(|\psi^0(1,2\dots,\mathrm{A})\rangle + |\psi^{res}(1,2\dots,\mathrm{A})\rangle\right) =$$
$$(E^0 + \mathrm{E}^{res})\left(|\psi^0(1,2\dots,\mathrm{A})\rangle + |\psi^{res}(1,2\dots,\mathrm{A})\rangle\right) \qquad \dots (6)$$

Then simply to.

$$\widehat{H}^{(o)}|\psi^0(1,2\dots,\mathrm{A})\rangle = E^0|\psi^0(1,2\dots,\mathrm{A})\rangle \qquad \dots (7)$$

$$\widehat{H}^{(o)}|\psi^{res}(1,2\dots,\mathrm{A})\rangle + \widehat{H}_{res}^{(1)}|\psi^0(1,2\dots,\mathrm{A})\rangle = E^0|\psi^{res}(1,2\dots,\mathrm{A})\rangle$$
$$+\mathrm{E}^{res}|\psi^0(1,2\dots,\mathrm{A})\rangle \qquad \dots (8)$$

Multiply Eq.(8) by $|\psi^0(1,2\ldots,A)\rangle$ and reformation the result equation , we can gate.

$$E^{res}\langle\psi^0(1,2\ldots,A)|\psi^0(1,2\ldots,A)\rangle = \left\langle\psi^0(1,2\ldots,A)\middle|\hat{H}^{(1)}_{res}\middle|\psi^0(1,2\ldots,A)\right\rangle + \langle\psi^0(1,2\ldots,A)|\hat{H}^{(o)} - E^0|\psi^{res}(1,2\ldots,A)\rangle \quad \ldots(9)$$

Depending on Eq.(6) the second term of Eq.(9) vanishes:

when $\hat{H}^{(o)}$ is Hermition operator, then.

$$\langle E^{res}\rangle = \left\langle\psi^0(1,2\ldots,A)\middle|\hat{H}^{(1)}_{res}\middle|\psi^0(1,2\ldots,A)\right\rangle \quad \ldots(10)$$

The energy of state in Eq.(5)

$$E = E^0 + E^{res} = \langle\psi^0(1,2\ldots,A)|\hat{H}^{(o)}|\psi^0(1,2\ldots,A)\rangle + \left\langle\psi^0(1,2\ldots,A)\middle|\hat{H}^{(1)}_{res}\middle|\psi^0(1,2\ldots,A)\right\rangle \quad \ldots(11)$$

$$= \sum_{j=1}^{A} e_{ij} + \left\langle\psi^0(1,2\ldots,A)\middle|\hat{H}^{(1)}_{res}\middle|\psi^0(1,2\ldots,A)\right\rangle \quad \ldots(12)$$

Here $\sum_{j=1}^{A} e_{ij}$ is the single particle energies and $\left\langle\psi^0(1,2\ldots,A)\middle|\hat{H}^{(1)}_{res}\middle|\psi^0(1,2\ldots,A)\right\rangle$ is the residual interaction.

Due to shell model calculation one assume that a nuclei most be made in term of an inert core of closed shell and extra nucleons in the orbit not occupied by core nucleus. Then the total binding energies are given by Hadi[11].

$$E^B\,(\text{core} + \delta^2) = E_{JT} + E^{B.E}(core) \quad \ldots(13)$$

Where E_{JT} is the energy of the residual interaction that given by Amos *etal* [12].

$$E_{JT} = \sum_i \varepsilon_i + \sum_{a\le b,c\le d}\sum_{JT} V_{JT}(ab;cd)\hat{T}_{JT}(ab;cd) \quad \ldots(14)$$

here ε_i are the single-particle energies with quantum number i, $V_{JT}(ab;cd)$ is a two-body matrix element, and $\hat{T}_{JT}(ab;cd)$ is the scalar two-body transition density for nucleon pairs (a, b) and (c, d), each pair coupled to spin quantum numbers JM.

And $E^{B.E}(core)$ is the energy of the core that assume and E_{JT} is given by Paola[13].

$$E^{B.E}(core) = 935[Zm_H + Nm_n - M(A,Z \quad \ldots(15)$$

Then the total energies is written by

$$E^B\,(\text{core} + \delta^2) = \sum_i \varepsilon_i + \sum_{a\le b,c\le d}\sum_{JT} V_{JT}(ab;cd)\hat{T}_{JT}(ab;cd) + E^{B.E}(core) \quad \ldots(16)$$

is employed to account The two body, the residual interaction Energy with Wildenthal ,and the universal potential energies (USDA,and USDB)

is qualitatively for the mass dependence expected from the evaluation of a medium-range interaction with harmonic oscillator radial wave functions, as was done for the original USD. The mass dependence is of the form Percival[14]

$$V_{JT}(j_aj_b;j_cj_d)^A = (\tfrac{18}{A})^{0.3}\left(\left\langle j_aj_b\middle|V^{Int}_{1,2}\middle|j_cj_d\right\rangle_{JT=1}\right)^{A=18} \quad \ldots(17)$$

The excitation energy $E_{Ext\ (k)}$ of k th excited state follows from the binding energy of the nucleus in that state taken with results respect to the ground state binding energy given by Brussard and Glaudemans[15]

$$E_{Ext\ (k)} = E^{B.E}\,(k) - E^{B.E}\ (k) \qquad \dots (18)$$

RESULTS

Shell model based on the quantum theory have been used to study the nuclear structure depending on the calculated of the binding and excitation energies for ${}^{30}_{14}\mathrm{Si}_{16}$.

The binding and excitation energies is calculated depending on the formalism in Eq.(16)due to the two body matrix element and single partical energies for nuclei that assume to be described by an inert closed shell core${}^{28}_{14}\mathrm{Si}_{14}$ and two nucleons in S-D shell model. We take experimental single particals energy that find from difrent experimental energies levels $\varepsilon_{2S\frac{1}{2}} = E^{B.E}\,({}^{29}_{14}\mathrm{Si}_{15}) - E^{B.E}\,({}^{28}_{14}\mathrm{Si}_{14}) = \big(-245.02 - (-236.54)\big) = -8.48MeV$ and $\varepsilon_{2S\frac{1}{2}} - \varepsilon_{1d\frac{3}{2}} = 1.27\ MeV$[15]. The model space $\langle \mathbf{2S_{1/2}1d_{3/2}} \rangle$ describe in the representation **(J^{π} ,T)** combination **(0^+,1)** and **(1^+,0)** for $|\,\mathbf{2S_{1/2}}\rangle$, **(2^+,1), (2^+,0)** , **(1^+,0)** , **(1^+,1)** and **(0^+,1)** for $|\,\mathbf{2S_{1/2}\ 1d3/2}\ \rangle$ and **(3^+,0)** , **(2^+,1)** , **(1^+,0)** and**(0^+,1)** for $|\,\mathbf{1d_{3/2}}\rangle$ and allowed for the two neutrons in configuration space.

The matrix element of two particles interaction are calculated using Eq(17)with a MATLAB program and result of our calculation are listed in **Table(1).**

The binding energies of the core are calculated from Eq(15) with mass number are M(${}^{28}_{14}\mathrm{Si}_{14}$) = $(28.976494700\ )\boldsymbol{amu}$,**M($_0$n^1)=(1.008665)amu and M(p)=(1.007825)amu for** ,neutron and proton respectively Audi[16].

Binding energies of the ${}^{30}_{14}\mathrm{Si}_{16}$ nuclease for each term for configuration (J^{π} ,T) values can be evaluated from Eq(16),the result are shown in **Table(2).** Therefore the excitation energies follow directly from the different values and calculated using the Eq(18),results are summarized in **Table(3)**.

Table-(1): Results of our calculation of the matrix element for Wildenthal, USDA, and USDB interaction.

J_a	J_b	J_c	J_d	J	T	$\langle j_a j_b \lvert V^W \rvert j_c j_d \rangle$	$\langle j_a j_b \lvert V^{USDA} \rvert j_c j_d \rangle$ Percival[14]	$\langle j_a j_b \lvert V^{USDB} \rvert j_c j_d \rangle$ Percival[14]
1/2	**1/2**	**1/2**	**1/2**	**0**	**1**	**-2.1246**	**-1.8461**	**-1.6913**
1/2	**1/2**	**1/2**	**1/2**	**1**	**0**	**-3.2628**	**-3.8693**	**-3.7093**
1/2	**3/2**	**1/2**	**3/2**	**2**	**1**	**-0.4064**	**-0.2533**	**-0.3034**
1/2	**3/2**	**1/2**	**3/2**	**2**	**0**	**-1.8194**	**-1.753**	**-1.8504**
1/2	**3/2**	**1/2**	**3/2**	**1**	**0**	**-4.2930**	**-3.8051**	**-4.0460**
1/2	**3/2**	**1/2**	**3/2**	**1**	**1**	**0.6066**	**0.3105**	**0.5158**
1/2	**3/2**	**1/2**	**3/2**	**0**	**1**	**-1.0835**	**-0.9834**	**-1.0150**
3/2	**3/2**	**3/2**	**3/2**	**3**	**0**	**-0.4000**	**-2.9800**	**-2.9660**
3/2	**3/2**	**3/2**	**3/2**	**2**	**1**	**-0.0665**	**-0.1570**	**-0.0974**
3/2	**3/2**	**3/2**	**3/2**	**1**	**0**	**-1.4151**	**-1.4927**	**-1.6582**
3/2	**3/2**	**3/2**	**3/2**	**0**	**1**	**-2.1845**	**-1.505**	**-1.8992**

Table-(2): Results of the binding energy B.E (MeV) for ${}^{30}_{14}Si_{16}$ that calculated with Wildenthal,USDA,and USDB.

Configuration	J^{π} T	Binding Energy (B.E)(Mev)		
		Wildenthal	USDA	USDB
$(2S1/2)^2$	0+ 1	-255.3227309	-255.0838009	-254.9509900
$(2S1/2)^2$	1+ 0	-256.2992122	-256.8195390	-256.6822723
$2S_{3/2}\,1d_{3/2}$	2+ 1	-252.5786576	-252.4473104	-252.4902921
$2S_{3/2}\,1d_{3/2}$	2+ 0	-253.7908946	-253.7339289	-253.8174900
$2S_{3/2}\,1d_{3/2}$	1+ 1	-255.9130385	-255.4944607	-255.7011330
$2S_{3/2}\,1d_{3/2}$	1+ 0	-251.7095874	-251.9636167	-251.7874863
$2S_{3/2}\,1d_{3/2}$	0+ 1	-253.1595533	-253.0736758	-253.1007860
$(1d\,3/2)^2$	3+ 0	-251.3031669	-253.5165933	-253.5045824
$(1d\,3/2)^2$	2+ 1	-251.0170515	-251.0946930	-251.0435611
$(1d\,3/2)^2$	1+ 0	-252.1740386	-252.2406130	-252.3825983
$(1d\,3/2)^2$	0+ 1	-252.8341201	-252.2511654	-252.5893563

Table-(3): Results of the excitation energies E.X (MeV) for ${}^{30}_{14}Si_{16}$that calculated with Wildenthal,USDA,and USDB interaction.

Configuration	J^{π} T	Excitation Energy (Ex)(Mev)		
		Wildenthal	USDA	USDB
$(2S1/2)^2$	0+ 1	0.00000000	0.00000000	0.0000000
$(2S1/2)^2$	1+ 0	-0.97648713	-1.7357381	-1.7312769
$2S_{3/2}\,1d_{3/2}$	2+ 1	2.7440737	2.6364905	2.4607033
$2S_{3/2}\,1d_{3/2}$	2+ 0	1.5318363	1.3498720	1.1335054
$2S_{3/2}\,1d_{3/2}$	1+ 1	0.5903076	-0.4106598	-0.7501376
$2S_{3/2}\,1d_{3/2}$	1+ 0	3.6131435	3.1201842	3.1635091
$2S_{3/2}\,1d_{3/2}$	0+ 1	2.1631776	2.0101251	1.8502094
$(1d\,3/2)^2$	3+ 0	4.0195640	1.5672077	1.4464130
$(1d\,3/2)^2$	2+ 1	4.3056794	3.9891079	3.9074343
$(1d\,3/2)^2$	1+ 0	3.1486923	2.8431879	2.5683971
$(1d\,3/2)^2$	0+ 1	2.4886108	2.8326355	2.3616391

Table-(4): Compared our results of the binding and excitation energies (MeV) for ${}^{30}_{14}Si_{16}$that calculated with Wildenthal, USDA, and USDB with expermental data.

Configuration	J^{π} T	Binding Energy (B.E)(Mev)			Expermental
		-255.3227309	-255.0838009	-254.9509900	-255.6000000[17]
		Excitation Energy (Ex)(Mev)			
		Wildenthal	USDA	USDB	
$(2S1/2)^2$	0+ 1	0.00000000	0.00000000	0.0000000	
$(2S1/2)^2$	1+ 0	-0.97648713	-1.7357381	-1.7312769	
$2S_{3/2}$ 1d $_{3/2}$	2+ 1	2.7440737	2.6364905	2.4607033	2.24
$2S_{3/2}$ 1d $_{3/2}$	2+ 0	1.5318363	1.3498720	1.1335054	
$2S_{3/2}$ 1d $_{3/2}$	1+ 1	0.5903076	-0.4106598	-0.7501376	
$2S_{3/2}$ 1d $_{3/2}$	1+ 0	3.6131435	3.1201842	3.1635091	3.77
$2S_{3/2}$ 1d $_{3/2}$	0+ 1	2.1631776	2.0101251	1.8502094	
$(1d\ 3/2)^2$	3+ 0	4.0195640	1.5672077	1.4464130	
$(1d\ 3/2)^2$	2+ 1	4.3056794	3.9891079	3.9074343	3.5
$(1d\ 3/2)^2$	1+ 0	3.1486923	2.8431879	2.5683971	
$(1d\ 3/2)^2$	0+ 1	2.4886108	2.8326355	2.3616391	3.79

DISCUSSION

The shell model based on the wave functions of the configuration model was applied to analyze the theoretical data of the low-lying level structure sd-shell nuclei ^{28,30}Si and nucleon interaction with them a unified way.

It is shown that the sequence of the lower levels is well, but that the level spacing is somewhat tool the effect of variation of the strength parameter for calculation in USDA, and USDB compare with less in Wildenthal.

Furthermore, the excitation energies that calculations theoretically are quite for first or second excited state but it's become very complicated for higher spectral because several nucleons can be excited simultaneously into super position of many different configuration to produce a given nuclear spin and parity. The ground state configuration indicate that all the proton sub shell filled, and all the neutron is effect on the excitation

The Wildenthal, and USDA, was found to be quite successful due with USDB in describing the levels structure up to excitation energy for sd-shell nuclei comparing with experimental. On the other hand from table (4), we are looking for the low-lying state that agreement with experimental data.

CONCLUSION

In summary, we have applied the shell model to study the binding and excitation energies for${}^{30}_{14}\boldsymbol{Si}$, using the Wildenthal, USDA,and. USDB. We found a Wildenthal and USDA potential that describe the interaction between two body in the configuration is good agreement with the experimental data results compare the potential USDB .

Generally, the Wiesenthal USDA, and USDB with experimental single energies for S ans D shell enable us to describe the nuclear structure for light nucles in S-D shell.In our calculation results , the effect of residual interaction force emergesevidently when we compare the calculated for three potential with experimental data

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