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The Effect of Rotation on the Onset of Double Diffusive Convection in a Saturated Anisotropic Porous Layer with Internal Heat Source.

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Abstract: In the present study, the effect of rotation on the onset of convection in a saturated anisotropic porous layer with internal heat source is investigated analytically using linear and nonlinear theories. The linear theory is based on the normal mode technique and the nonlinear theory on the truncated Fourier series. The Brinkman model that includes the Coriolis term is employed for the momentum equation. The onset criterion for stationary, oscillatory and finite amplitude convection is derived analytically. The effect of internal Rayleigh number, Taylor number, Darcy-Prandtl number, Thermal and Mechanical anisotropic parameters, Lewis number, normalized porosity and solute Rayleigh number on stability of a system is shown graphically. The transient behavior of the Nusselt and Sherwood numbers is obtained by solving the finite amplitude equations using the Runge-Kutta method.

Keywords: Double Diffusive Convection, Rotation, Brinkman model, Porous layer, Anisotropy, Internal Heat Source, Heat and Mass Transfer.

INTRODUCTION

The problem of convection induced by temperature and concentration gradients or by concentration gradients of two species, known as double diffusive convection, has attracted considerable interest in the last several decades. If gradients of two stratifying agencies having different diffusivities are simultaneously present in a fluid layer, a variety of interesting convective phenomena can occur that are not possible in single component fluids. The double diffusive convection in porous media has also become important in recent years because of its many applications in geophysics, particularly in saline geothermal fields where hot brines remain beneath less saline, cooler ground waters. A comprehensive review of the literature concerning double diffusive convection in a binary fluid-saturated porous medium may be found in the book by Nield and Bejan[1]. Excellent review articles on double diffusive convection in porous media include those by Trevisan and Bejan[2], Mojtabi and Charrier–Mojtabi[3, 4] and Mamou[5].

Early studies on the phenomena of double diffusive convection in porous media are mainly concerned with problem of convective instability in a horizontal layer heated and salted from below. The previous studies have usually been concerned with homogeneous isotropic porous structures. However, during the last few years the effect of nonhomogeneity and anisotropy of porous medium has been studied. The geological and pedagogical processes rarely form isotropic medium as is usually assumed in transport studies. Processes such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of a naturally anisotropic porous medium. Anisotropy can also be a characteristic of manufactured porous media like those made of irregularly shaped particles formed by extrusion or pelleting used in chemical engineering process or fiber materials used for insulating purposes.

Natural convection heat transfer induced by internal heat generation has recently received considerable attention because of numerous applications in geophysics and energy-related engineering problems. Such applications include heat removal from nuclear fuel debris, underground disposal of radioactive waste materials, storage of foodstuff, and exothermic chemical reactions in packed-bed reactor. Acharya and Goldstein[6] studied numerically a complicated inclined cavity with inner heat generation. Mostly internal heating and (at least if one assumes whole mantle convection) with only the small bottom heating that arises from core cooling. As a consequence, plumes are a relatively unimportant part of the convecting system. Many authors have considered the conditions for instability in a porous layer heated either from below or by means of internal volumetric heat generation. Horton and Rogers[7], and Lapwood[8] were the first to establish analytically the critical Rayleigh number for the onset of convection in a fluid-saturated porous layer heated from below without internal heat generation. Their analysis has since been extended substantially to include other types of modeling of porous media, and to moderately and strongly nonlinear situations. The reviews by Rees[9] and Tyvand[10] may be consulted for further details. Stability and bifurcation were studied numerically for steady internal heating of a horizontal layer cooled from above and below by Marimbordes *et al.* [11] and for a vertical channel by Nagata and Generalis[12]. However, there are relatively very few studies available in which the effect of internal heating on convective flow has been investigated. Some of these studies are Herron[13], Borujerdi *et al.* [14], Khalili *et al.* [15], Hill[16], Mansour et al. [17], Cookey *et al.* [18], Kannan and Venkataraman[19], Cookey and Omubo- Pepple[20], and Khan and Aziz[21]. More recently Bhadauria [22] has investigated double-diffusive convection in a saturated anisotropic porous layer with an internal heat source by employing a generalized Darcy model for the momentum equation and extended work is seen in Bhadauria *et al.* [23]

Double diffusive convection occurs in many systems in industry and nature, and in the present context, is of particular interest in the study of extraction of metals from ores where a mushy layer is formed during solidification of a metallic alloy. Further, the quality and structure of the resulting solid can be controlled by influencing the transport process externally, which can be done by rotation or by internal heating. However in the present study, internal heating of the system was used as an external means to influence the transport process, thereby controlling the quality and structure of the resulting solid. Further, many of the previous studies have modeled the mushy layer as isotropic porous medium, however in reality, the permeability of the porous medium is anisotropic as discussed above.

MATHEMATICAL FORMATION

We consider an infinite horizontal anisotropic porous layer confined between the plates $z=0$ and $z=d$, with vertically downward gravity $\mathbf{g}$ acting on it. A uniform adverse temperature gradient $\Delta T = T_l - T_u$ and a stabilizing concentration gradient $\Delta S = S_l - S_u$ $(T_l > T_u \; and \; S_l > S_u)$ are maintained between the lower and upper surfaces. The porous layer rotates uniformly about the z-axis with a constant angular velocity $\mathbf{\Omega} = (0,0,\Omega)$. A Cartesian frame of reference is chosen with origin in the lower boundary and z-axis vertically upwards. The interaction between heat and mass transfer known as the Soret and Dufour effects, is supposed to have no influence on the convective flow, so they are ignored. It is also assumed that the fluids and solid phase are in local thermal equilibrium. The Darcy –Brinkman model is employed for the momentum equation (see Zhang *et al.* [24]). The velocities are assumed to be small so that the advective and Forchheimer inertia effected are ignored. The Boussinesq approximation, which states that the variation in density is negligible everywhere in the conservations except in the buoyancy term, is assumed to hold, with these assumptions the basic governing equations are

$$\nabla.\mathbf{q} = 0, \tag{3.1}$$

$$\rho_0\left(\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{2}{\varepsilon}\,\mathbf{\Omega}\times\mathbf{q}\right) = -\nabla p + \rho\,\mathbf{g} - \mu\mathbf{K}.\mathbf{q}, \tag{3.2}$$

$$\gamma\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = \kappa_T\,\nabla^2 T + Q\left(T - T_0\right), \tag{3.3}$$

$$\varepsilon\frac{\partial S}{\partial t} + (\mathbf{q}.\nabla)S = (\kappa_S\,\nabla^2 S), \tag{3.4}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)], \tag{3.5}$$

where the variables and constants have their usual meaning as given in the nomenclature. Further, $\gamma = \dfrac{(\rho c)_{\mathrm{m}}}{(\rho c_p)_f}, (\rho c)_m = (1-\varepsilon)(\rho c_S) + \varepsilon\left(\rho C_p\right)_f$, c_p is the specific heat of the fluid at constant pressure, c is the specific heat of the solid, the subscripts f, s and m denote fluid, solid and porous medium values respectively.

Basic State: The basic state is assumed to be quiescent and is given by

$$\mathbf{q}_b = (0,0,0),\ \rho = \rho_b(z),\ p = p_b(z),\ T = T_b(z),\ S = S_b(z). \tag{3.6}$$

The temperature $T_b(z)$, solute concentration $S_b(z)$, pressure $p_b(z)$ and density $\rho_b(z)$, satisfy the following equations

$$\frac{dp_b}{dz} = -\rho_b\, g,\ \kappa_T\frac{d^2T}{dz^2} + Q\left(T_b - T_0\right) = 0, \quad \frac{d^2 S_b}{dz^2} = 0, \tag{3.7}$$

$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - S_0)]. \tag{3.8}$$

Then the basic state temperature and concentration are given by

$$\left\{\begin{aligned} T_b(z) &= \Delta T \; Sin\,\frac{\left(\sqrt{Ri}\left(1-\frac{z}{d}\right)\right)}{Sin\sqrt{Ri}}+T_0 \\ S_b &= \Delta S\left(1-\frac{z}{d}\right)+S_0 \end{aligned}\right\}, \tag{3.9}$$

where $Ri = \dfrac{Qd^2}{\kappa_{T\,z}}$ is the internal Rayleigh number.

Perturbed State: Now superimpose the small perturbations at the basic state in the form

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}',\; T = T_b(z) + T',\, S = S_b(z) + S', \tag{3.10}$$
$$p = p_b(z) + p',\;\; \rho = \rho_b(z) + \rho',$$

where the prime indicates that the quantities are infinitesimal perturbations. Substituting Eq. (3.10) into Eqs. (3.1)- (3.5) and using the basic state solutions, we obtain the equations governing the perturbations in the form

$$\nabla.\mathbf{q}' = 0, \tag{3.11}$$

$$\rho_0\left(\frac{1}{\varepsilon}\frac{\partial \mathbf{q}'}{\partial t}+\frac{2}{\varepsilon}\,\mathbf{\Omega}\times\mathbf{q}'\right) = -\nabla p' + \rho'\mathbf{g} - \mu\,\mathbf{K}.\mathbf{q}', \tag{3.12}$$

$$\gamma\frac{\partial T'}{\partial t}+(\mathbf{q}'.\nabla)T' - w'\frac{\Delta T}{d} = \kappa_T\,\nabla^2 T' + QT', \tag{3.13}$$

$$\varepsilon\frac{\partial S'}{\partial t}+(\mathbf{q}'.\nabla)S' - w'\frac{\Delta S}{d} = \left(\kappa_S\,\nabla^2 S'\right), \tag{3.14}$$

$$\rho' = -\rho_0\left(\beta_T T' + \beta_S S'\right). \tag{3.15}$$

by operating curl twice on equation (3.12), we eliminate p' from it and then render the resulting equation and Eqs. (3.13) and (3.14) dimensionless using the following transformations

$$(x',y',z') = d\left(x^*,y^*,z^*\right),\; t' = \left(\frac{\gamma d^2}{\kappa_{Tz}}\right)t^*,\; (u',v',w') = \frac{\kappa_{Tz}}{d}(u^*,v^*,w^*), \tag{3.16}$$
$$T' = (\Delta T)T^*,\; S' = (\Delta S)S^*,$$

to obtain non-dimensional equations as (after dropping the asterisks for simplicity)

$$\left(\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2+\nabla_h^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}+\frac{1}{\xi}\right)+Ta\frac{\partial^2}{\partial z^2}\right)w = \left(\frac{1}{Pr_D}\frac{\partial}{\partial t}+\frac{1}{\xi}\right)\left(Ra_T\nabla_h^2 T - Ra_S\nabla_h^2 S\right), \tag{3.17}$$

$$\left(\frac{\partial}{\partial t}-\left(\eta\nabla_h^2+\frac{\partial^2}{\partial z^2}\right)+(\vec{q}.\nabla)-Ri\right)T - w = 0, \tag{3.18}$$

$$\left(\phi\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2+(\vec{q}.\nabla)\right)S-w=0, \tag{3.19}$$

The dimensionless groups, that appear are $\Pr_D=\gamma\varepsilon\nu d^2/K_z\kappa_{Tz}$ the Darcy-Prandtl number, $Ta=(2\Omega K_z/\varepsilon\nu)^2$ the Taylor number, $Ra_T=\beta_T g\Delta T d K_{Tz}/\nu\kappa_{Tz}$ the thermal Rayleigh number, $Ra_S=\beta_S g\Delta S d K_{Tz}/\nu\kappa_{Tz}$ the solute Rayleigh number, $Ri=Qd^2/\kappa_{Tz}$ the Internal Rayleigh number, $Le=\kappa_T/\kappa_s$ the Lewis number, $\xi=k_x/k_z$ the mechanical anisotropy parameter, $\eta=\kappa_{Tx}/\kappa_{Tz}$ is the thermal anisotropy parameter, $\phi=\varepsilon/\gamma$ normalized porosity. Eqs. (3.17)-(3.19) are solved for stress free, isothermal and isosolutal boundary conditions. Hence the boundary conditions for the perturbation variables are given by

$$w=\frac{\partial^2 w}{\partial z^2}=T=S=0,\ \text{at}\quad z=0,1. \tag{3.20}$$

The stress-free boundary conditions are chosen for mathematical simplicity without qualitatively important physical effect being lost. The use of stress-free boundary condition is a useful mathematical simplification but is not physically sound. The correct boundary condition for a viscous binary fluid are to impose rigid rigid boundary condition but then the problem is not tractable analytically.

LINEAR STABILITY ANALYSIS

In this section we predict the thresholds of both marginal and oscillatory convections using linear theory. The Eigenvalue problem defined by Eqs. (3.17)- (3.19) subject to the boundary conditions (3.20) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that amplitudes of the perturbation are very small we write

$$\begin{pmatrix} w\\ T\\ S\end{pmatrix}=\begin{pmatrix} W(z)\\ \Theta(z)\\ \Phi(z)\end{pmatrix}\exp\left[i(lx+my)+\sigma t\right], \tag{3.21}$$

Where l and m are the wave numbers in the horizontal plane and σ is the growth rate. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter σ. Substituting Eq. (3.21) into Eqs. (3.17)- (3.19), we obtain

$$\left(\left[\frac{\sigma}{\Pr_D}(D^2-a^2)+\left(\frac{D^2}{\xi}-a^2\right)\right]\left(\frac{\sigma}{\Pr_D}+\frac{1}{\xi}\right)+TaD^2\right)W=\left(\frac{\sigma}{\Pr_D}+\frac{1}{\xi}\right)\left(-a^2Ra_T\Theta+a^2Ra_S\Phi\right), \tag{3.22}$$

$$\left[\sigma-(D^2-a^2\eta)-Ri\right]\Theta-W=0, \tag{3.23}$$

$$\left[\phi\sigma-\frac{1}{Le}(D^2-a^2)\right]\Phi-W=0, \tag{3.24}$$

where $D=d/dz$ and $a^2=l^2+m^2$.

We assume the solution of Eqs. (3.22)- (3.24) satisfying the boundary conditions (3.20) in the form,

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \sin n\pi z, (n = 1, 2, 3........). \tag{3.25}$$

The most unstable mode corresponding to $n=1$ (fundamental mode). Therefore, substituting Eq (3.25) with $n=1$ into Eqs. (3.22)- (3.24), we obtain a matrix equation of the form

$$\begin{pmatrix} M_{11} & -a^2 Ra_T & a^2 Ra_S \\ -1 & \sigma + \delta_2{}^2 - Ri & 0 \\ -1 & 0 & \phi\sigma + \dfrac{1}{Le}\delta^2 \end{pmatrix} \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{3.26}$$

Where $M_{11} = \dfrac{\sigma}{\Pr_D}\delta^2 + \delta_1{}^2 + \pi^2 Ta\left(\sigma \Pr_D{}^{-1} + \xi^{-1}\right)^{-1}$, $\delta^2 = \pi^2 + a^2$, $\delta_1^2 = \pi^2 \xi^{-1} + a^2$ and $\delta_2^2 = \pi^2 + \eta a^2$

The conditions of non-trivial solutions of above system of homogenous linear equations (3.26) yields the expression for the thermal Rayleigh number in the form

$$Ra_T = \left(\frac{\sigma + \delta_2{}^2 - Ri}{a^2}\right)\left[\frac{\sigma\delta^2}{\Pr_D} + \delta_1^2 + \frac{\pi^2 Ta}{\left(\sigma \Pr_D{}^{-1} + \xi^{-1}\right)} + \frac{a^2 Ra_S}{\phi\sigma + Le^{-1}\delta^2}\right]. \tag{3.27}$$

Marginal State

For validity of the principle of exchange of stabilities (i.e. steady case), we have $\sigma = 0$ (i.e. $\sigma_r = \sigma_i = 0$) at the margin of stability. Then the Rayleigh number at which marginally stable steady mode exists becomes

$$Ra_T{}^{st} = \left(\frac{\eta a^2 + \pi^2 - Ri}{a^2}\right)\left(\frac{\pi^2}{\xi} + a^2 + \pi^2 Ta\,\xi\right) + \frac{\left(\eta a^2 + \pi^2 - Ri\right) Le\, Ra_S}{\left(\pi^2 + a^2\right)}. \tag{3.28}$$

The minimum value of the Rayleigh number Ra_T^{st} occurs at the critical wave number $a = a_c^{st}$.

In the absence of a heat source for the porous medium (i.e. when Ri = 0) Eq. (3.28) reduces to,

$$Ra_T{}^{st} = \left(\frac{\eta a^2 + \pi^2}{a^2}\right)\left(\frac{\pi^2}{\xi} + a^2 + \pi^2 Ta\,\xi\right) + \frac{\left(\eta a^2 + \pi^2\right) Le\, Ra_S}{\left(\pi^2 + a^2\right)}. \tag{3.29}$$

This is exactly the one given by Malashetty and Heera[25]. When $Ta \to 0$, i.e., in absence of rotation, Eq. (3.28) reduces to

$$Ra_T{}^{st} = \frac{1}{a^2}\left(\eta a^2 + \pi^2\right)\left(\frac{\pi^2}{\xi} + a^2\right) + \frac{\left(\eta a^2 + \pi^2\right) Le\, Ra_S}{\left(\pi^2 + a^2\right)}, \tag{3.30}$$

which is identical with Malashetty and Swamy [26]. Further, for an isotropic porous medium, that is, when $\xi = \eta = 1$, Eq. (3.28) gives

$$Ra_T{}^{st} = \frac{\pi^4\left(1+\alpha^2\right)^3}{\alpha^2}\eta_1 + \frac{Ta}{\alpha^2\eta_1} + Le\,Ra_S, \tag{3.31}$$

Where $\eta_1 = \frac{1}{\pi^4}\left(\pi^4 Da + \left(\pi^2+a^2\right)^{-1}\right)$, $\alpha^2 = \frac{a^2}{\pi^2}$, which is the one obtained by Rudraiah et al. [27].

Oscillatory State

We now set $\sigma = i\sigma$ in Eq. (3.27) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\sigma_i\Delta_2, \tag{3.32}$$

where Δ_1 and Δ_2 are given as

$$\Delta_1 = \frac{-\delta^2 w^2 + \delta_1^2 \Pr_D\left(\delta_2^2 - Ri\right)}{a^2 \Pr_D} + \frac{\pi^2 Ta \Pr_D \xi\left(w^2\xi + \Pr_D\left(\delta_2^2 - Ri\right)\right)}{a^2\left(\Pr_D{}^2 + \xi^2 w^2\right)} + \frac{Ra_S\left(\phi w^2 + \delta^2 Le^{-1}\left(\delta_2^2 - Ri\right)\right)}{\left(\delta^2 Le^{-1}\right)^2 + \phi^2 w^2}$$

$$\Delta_2 = \frac{\delta_1^2 \Pr_D + \delta^2\left(\delta_2^2 - Ri\right)}{a^2 \Pr_D} + \frac{\pi^2 Ta \Pr_D \xi\left(\Pr_D + \xi\left(\delta_2^2 - Ri\right)\right)}{a^2\left(\Pr_D{}^2 + \xi^2 w^2\right)} + \frac{Ra_S\left(\delta^2 Le^{-1} - \phi\left(\delta_2^2 - Ri\right)\right)}{\left(\delta^2 Le^{-1}\right)^2 + \phi^2 w^2}$$

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (3.32) it follows that either $\sigma_i = 0$ (steady onset) or $\Delta_2 = 0\ \left(\sigma_i \neq 0\right)$ (oscillatory onset).

For oscillatory onset $\Delta_2 = 0\ \left(\sigma_i \neq 0\right)$ and this gives an expression for frequency of oscillations in the form (on dropping the subscript i)

$$a_0\left(\sigma^2\right)^2 + a_1\left(\sigma^2\right) + a_2 = 0, \tag{3.33}$$

Where the coefficients are given as

$$a_0 = -LeRi\delta^2\xi^2\phi^2 + Le^2 \Pr_D \delta_1^2\xi^2\phi^2 + Le^2\delta^2\delta_2^2\xi^2\phi^2;$$

$$\begin{aligned} a_1 = {} & a^2 Le \Pr_D Ra_S \delta^2\xi^2 - Ri\delta^6\xi^2 + \Pr_D \delta^4\xi^2\delta_1^2 + \delta^6\xi^2\delta_2^2 + a^2 Le^2 \Pr_D Ra_S Ri\phi \\ & - a^2 Le^2 \Pr_D Ra_S \delta_2^2\xi^2\phi - Le^2 \Pr_D{}^2 Ri\delta^2\phi^2 + Le^2 \Pr_D{}^3 \delta_1^2\phi^2 + Le^2 \Pr_D{}^2 \delta^2\delta_2{}^2\phi^2 \\ & + Le^2\pi^2 \Pr_D{}^3 Ta\xi\phi^2 + Le^2\pi^2 \Pr_D{}^2 RiTa\xi^2\phi^2 - Le^2\pi^2 \Pr_D{}^2 Ta\delta_2{}^2\xi^2\phi^2; \end{aligned}$$

$$\begin{aligned} a_2 = {} & a^2 Le \Pr_D{}^3 Ra_S \delta^2 - \Pr_D{}^2 Ri\delta^6 + \Pr_D{}^3 \delta^4\delta_1^2 + \Pr_D{}^2 \delta^6\delta_2^2 \\ & + \pi^2 \Pr_D{}^3 Ta\delta^4\xi + \pi^2 \Pr_D{}^2 RiTa\delta^4\xi^2 - \pi^2 \Pr_D{}^2 Ta\delta^4\delta_2^2\xi^2 \\ & + a^2 Le^2 \Pr_D{}^3 Ra_S Ri\phi - a^2 Le^2 \Pr_D{}^3 Ra_S \delta_2^2\phi. \end{aligned}$$

Now Eq. (3.32) with $\Delta_2 = 0$ gives,

$$Ra_T^{\,osc} = \frac{1}{a^2}\left(\left(\delta_2{}^2 - Ri\right)\delta_1^2 - \frac{\sigma^2\delta^2}{\Pr_D}\right) + Ra_S\,\frac{\left(\delta^2 Le^{-1}\left(\delta_2{}^2 - Ri\right) + \sigma^2\phi\right)}{\left(Le^{-1}\delta^2\right)^2 + \sigma^2\phi^2}$$
$$+\pi^2\,Ta\,\frac{\left(\sigma^2\Pr_D{}^{-1} + \left(\delta_2{}^2 - Ri\right)\xi^{-1}\right)}{a^2\left(\left(\xi^{-1}\right)^2 + \left(\sigma\Pr_D{}^{-1}\right)^2\right)}. \tag{3.34}$$

WEAK NONLINEAR ANALYSIS

Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding Eigen functions describing qualitatively the convective flow, it cannot provide information about the values of the convection amplitudes, nor regarding the rate of heat transfer. To obtain this additional information, a weak nonlinear stability analysis is performed using a truncated Fourier series representation method.

For simplicity of analysis, we confine ourselves to the two-dimensional rolls, so that all the physical quantities are independent of *y*. We introduce a stream function such that $u = \partial\psi/\partial z,\;\; w = -\partial\psi/\partial x$ into the Eqs. (3.14) - (3.15) to obtain

$$\left[\frac{1}{\Pr_D}\frac{\partial}{\partial t}\nabla^2 + \frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right]\psi - Ta^{1/2}\frac{\partial V}{\partial z} + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0, \tag{3.35}$$

$$\left(\frac{1}{\Pr_D}\frac{\partial}{\partial t} + \frac{1}{\xi}\right)\frac{\partial V}{\partial t} + Ta^{1/2}\frac{\partial^2\psi}{\partial z^2} = 0, \tag{3.36}$$

$$\frac{\partial T}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)T - \frac{\partial(\psi, T)}{\partial(x,z)} + \frac{\partial\psi}{\partial x} - Ri\,T = 0, \tag{3.37}$$

$$\left(\phi\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)S - \frac{\partial(\psi, S)}{\partial(x,z)} + \frac{\partial\psi}{\partial x} = 0. \tag{3.38}$$

Here, V is the z-component of the vorticity vector called zonal velocity. The first effect of non-linearity is to distort the temperature and concentration fields through the interaction of ψ, T and also ψ, S. The distortion of these fields will correspond to a change in the horizontal mean, i.e., a component of the form $\sin(2\pi z)$ will be generated. Thus a minimal Fourier series which describes the finite amplitude free convection is given by

$$\psi = A(t)\sin(ax)\sin(\pi z), \tag{3.39}$$

$$T = B(t)\cos(ax)\sin(\pi z) + C(t)\sin(2\pi z), \tag{3.40}$$

$$S = D(t)\cos(ax)\sin(\pi z) + E(t)\sin(2\pi z), \tag{3.41}$$

$$V = F(t)\cos(ax)\sin(\pi z) + G(t)\sin(2\pi x), \tag{3.42}$$

where the amplitudes $A(t),\, B(t),\, C(t), D(t), E(t), F(t)\; and\; G(t)$ are to be determined from the dynamics of the system.

Substituting Eqs. (3.39)- (3.42) into the coupled nonlinear system of partial differential Eqs. (3.35)-(3.38) and equating the coefficients of like terms, we obtain the following non-linear autonomous system of differential equations

$$A = -\frac{Pr}{\delta^2}\left(\delta_1^{\,2} A - \pi Ta^{1\backslash 2} F + a\,Ra_T B - a\,Ra_S D\right), \tag{3.43}$$

$$B = -\left(a\,A + (\delta_2^{\,2} - Ri)B + \pi a\,AC\right), \tag{3.44}$$

$$C = -4\pi^2 C + \frac{\pi a}{2} AB + Ri\,C, \tag{3.45}$$

$$D = -\frac{1}{\phi}\left(a\,A + \frac{\delta^2}{Le} D + \pi\,a\,AE\right), \tag{3.46}$$

$$E = \frac{1}{\phi}\left(\frac{\pi a}{2} AD - \frac{4\pi^2}{Le} E\right), \tag{3.47}$$

$$F = \frac{\mathrm{Pr}_D}{\pi}\left(-\frac{\pi}{\xi} F - \pi^2\,Ta^{1/2}\,A\right). \tag{3.48}$$

The non-linear system of autonomous differential equations is not suitable to analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. After determining the numerical values of the amplitudes $A(t),\ B(t),\ C(t), D(t), E(t), F(t)\ and\ G(t)$ the Nusselt and Sherwood numbers can be obtained as a function of time. However, one can make qualitative predictions as discussed below. The steady analysis is performed by setting the left-hand side of Eqs. (3.43) - (3.48) to zero.

Steady Finite Amplitude Motions

From qualitative predictions we look into possibility of an analytical solution. In case of steady motions, Eqs (3.43) - (3.48) can be solved in closed form. Setting the left-hand side of Eqs. (3.43) - (3.48) equal to zero, we get

$$\delta_1^{\,2} A - \pi Ta^{1\backslash 2} F + a\,Ra_T B - a\,Ra_S D = 0, \tag{3.49}$$

$$a\,A + \left(\delta_2^{\,2} - Ri\right)B + \pi a AC = 0, \tag{3.50}$$

$$4\pi^2 C - \frac{\pi a}{2} AB - Ri\,C = 0, \tag{3.51}$$

$$a\,A + \frac{\delta^2}{Le} D + \pi\,a\,AE = 0, \tag{3.52}$$

$$\frac{4\pi^2}{Le} E - \frac{\pi a}{2} AD = 0, \tag{3.53}$$

$$-\frac{\pi}{\xi}F-\pi^2 Ta^{1/2}A=0, \tag{3.54}$$

$$G=0, \tag{3.55}$$

writing B, C, D, E, F and G in terms of A, using Eqs. (3.50)-(3.55) and substituting these in Eq. (3.49) with $\frac{A^2}{8}=x$, we get

$$A_1x^2+A_2x+A_3=0, \tag{3.56}$$

Where $A_1,\ A_2\ and\ A_3$ are given as

$$A_1=4a^4Le^2\pi^2\delta_1^2+4a^4Le^2\pi^4Ta\delta_1^2\xi;$$
$$\begin{aligned}A_2&=4a^4Le\pi^2Ra_S-a^4Le^2Ra_T\left(4\pi^2-Ri\right)-4a^2Le^2\pi^2Ri\delta_1^2+4a^2\pi^2\delta^2\delta_1^2\\&+4a^2Le^2\pi^2\delta_1^2\delta_2^2+a^2Le^2Ri\delta_1^2\left(Ri-\delta_2^2\right)-4a^2Le^2\pi^4RiTa\delta_1^2\xi+4a^2\pi^4Ta\delta^2\delta_1^2\xi\\&+4a^2Le^2\pi^4Ta\delta_1^2\delta_2^2\xi+a^2Le^2\pi^2RiTa\delta_1^2\left(Ri-\delta_2^2\right)\xi;\end{aligned}$$
$$\begin{aligned}A_3&=-4a^2Le\pi^2Ra_SRi-a^2Ra_T\left(4\pi^2-Ri\right)\delta^2-4\pi^2Ri\delta^2\delta_1^2+4a^2Le\pi^2Ra_S\delta_2^2\\&+4\pi^2\delta^2\delta_1^2\delta_2^2+a^2LeRa_SRi\left(Ri-\delta_2^2\right)+Ri\delta^2\delta_1^2\left(Ri-\delta_2^2\right)-4\pi^4RiTa\delta^2\delta_1^2\xi\\&+4\pi^4Ta\delta^2\delta_1^2\delta_2^2\xi+\pi^2RiTa\delta^2\delta_1^2\left(Ri-\delta_2^2\right)\xi.\end{aligned}$$

The required root of Eq. (3.56) is given by

$$x=\frac{1}{2A_1}\left(-A_2+\left(A_2^{\,2}-4A_1A_3\right)^{\frac{1}{2}}\right). \tag{3.57}$$

When we let the radical in the above equation vanish, we obtain the expression for the finite amplitude Rayleigh number Ra^F, which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form

$$Ra^F=\frac{1}{2B_1}\left(-B_2+\left(B_2^{\,2}-4B_1B_3\right)^{\frac{1}{2}}\right), \tag{3.58}$$

Where $B_1,\ B_2\ and\ B_3$ are given as

$$\begin{aligned}B_1&=\left(16a^4Le^4\pi^4-8a^4Le^4\pi^2Ri+a^4Le^4Ri^2\right)\\&+8a^2Le\pi^2Ra_S\delta_1^2\left(4\pi^2\delta^2+Le^2\left(4\pi^2-Ri\right)\left(Ri-\delta_2^2\right)\right)\left(1+\pi^2Ta\xi\right)\\&+\delta_1^4\left(4\pi^2\delta^2+Le^2\left(4\pi^2-Ri\right)\left(Ri-\delta_2^2\right)\right)^2\left(1+\pi^2Ta\xi\right)^2;\end{aligned}$$
$$\begin{aligned}B_2&=\left(-32a^4Le^3\pi^4Ra_S+8a^4Le^3\pi^2Ra_SRi+8a^2Le^2\pi^2\delta_1^2\left(4\pi^2\delta^2+Le^2\left(4\pi^2-Ri\right)\left(Ri-\delta_2^2\right)\right)\left(1+\pi^2Ta\xi\right)\right)\\&-2a^2Le^2Ri\delta_1^2\left(4\pi^2\delta^2+Le^2\left(4\pi^2-Ri\right)\left(Ri-\delta_2^2\right)\right)\left(1+\pi^2Ta\xi\right);\end{aligned}$$
$$B_3=16a^4Le^2\pi^4Ra_S^{\,2}.$$

Heat and Mass Transport

In the study of convection in fluids, the quantification of the heat and the mass transport is important. This is because the onset of convection, as the Rayleigh number is increased, is more readily detected by its effect on the heat and mass transport. In the basic state, heat and mass transport is by conduction alone.

If H and J are the rate of heat and mass transport per unit area respectively, then

$$H=-\kappa_T\left\langle\frac{\partial T_{total}}{\partial z}\right\rangle_{z=0} \quad \text{and} \quad J=-\kappa_S\left\langle\frac{\partial S_{total}}{\partial z}\right\rangle_{z=0}, \tag{3.59}$$

Where the angular bracket corresponds to a horizontal average and

$$T_{total}=T_0-\Delta T\frac{z}{d}+T\left(x,z,t\right) \text{ and } S_{total}=S_0-\Delta S\frac{z}{d}+S\left(x,z,t\right), \tag{3.60}$$

Substituting Eq. (3.40) into (3.42) and using the resultant equation in Eq. (3.59), we get

$$H=\frac{\kappa_T\,\Delta T}{d}\left(1-2\pi\,C\right) \qquad \text{and} \qquad J=\frac{\kappa_S\,\Delta S}{d}\left(1-2\pi\,E\right). \tag{3.61}$$

The Nusselt (Nu) and Sherwood (Sh) numbers are respectively defined by

$$Nu=\frac{H}{\kappa_T\,\Delta T/d}=1-2\pi\,C, \tag{3.62}$$

$$Sh=\frac{J}{\kappa_S\,\Delta S/d}=1-2\pi E, \tag{3.63}$$

Writing C & E in terms of A, using Eqs (3.51)-(3.54) and substituting in Eqs.(3.62)-(3.63) respectively, we obtain

$$Nu=1+\frac{8\pi^2a^2x}{\left(\left(\delta_2^{\,2}-Ri\right)\left(4\pi^2-Ri\right)+4\pi^2a^2x\right)}, \tag{3.64}$$

$$Sh=1+\frac{2Le^2a^2x}{\left(\delta^2+Le^2a^2x\right)}. \tag{3.65}$$

RESULT AND DISCUSSION

The effect of rotation on the onset of double diffusive convection in a saturated anisotropic porous layer, in the presence of an internal heat source is investigated analytically using both linear and nonlinear theories. In the linear stability theory the expressions for the stationary and oscillatory Rayleigh number are obtained analytically along with the expressions for frequency of oscillation. The critical Rayleigh number for the oscillatory mode is derived as a function of internal Rayleigh number, solute Rayleigh number, normalized porosity, Darcy-Prandtl number, Taylor number and Lewis number. The nonlinear theory provides the quantification of heat and mass transport and also explains the possibility of the finite amplitude motions. The neutral stability curves in the Ra_T-a plane for various parameter values are as shown in Figs. 1-8. We fixed the values for the parameters as:

$$\xi = 0.3,\ \eta = 0.9,\ Ta = 100,\ Ra_S = 60,\ Le = 10,\ \phi = 0.9,\ \Pr_D = 50\ and\ Ri = 3$$

except the varying parameter, from these figures it is clear that the neutral curves are connected in a topological sense. This connection allows the linear stability criteria to be expressed in terms of the critical Rayleigh number Ra_{Tc}, below which the system is stable and unstable above.

We observe from the **figures 1(a) & (b),** that the effect of increasing Ri is to decrease the critical value of the Rayleigh number and corresponding wave number for stationary and oscillatory modes. Thus Ri has a destabilizing effect of rotation on the double diffusive convection in a saturated anisotropic porous layer.

Figures 2(a) & (b), 3(a) & (b) and 4(a) & (b) indicate the effect of Taylor number Ta, mechanical anisotropy parameter ξ and thermal anisotropy parameter η respectively on the neutral stability curves for the fixed values. It is observed that the critical values of stationary and oscillatory Rayleigh number increase with an increase of Ta, ξ and η, indicating that their effect is to inhibit the onset of both stationary and oscillatory convection. Thus Ta, ξ and η have a stabilizing effect on the system for stationary and oscillatory modes.

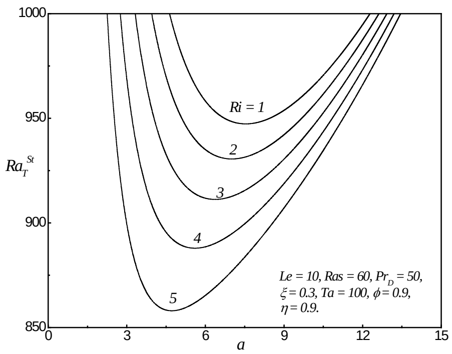


Fig 1(a). Stationary neutral stability curves for different values of an Internal Rayleigh number *Ri*.

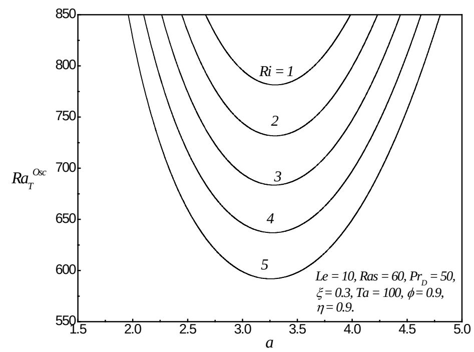


Fig 1(b). Oscillatory neutral stability curves for different values of an Internal Rayleigh number *Ri*.

The neutral stability curves for different values of normalized porosity ϕ are presented in **Fig. 5.** We observe from this figure that with an increase of ϕ the minimum oscillatory Rayleigh number decreases implying that the effect is to advance the onset of oscillatory convection. As normalized porosity increases, the thermal "lag" effect (double advective behavior in the terminology is reduced. This makes advective heat transfer more effective and so makes it easier for the destabilizing thermal buoyancy gradient to produce convection.

Fig. 6 exhibits the effect of Darcy-Prandlt number $\Pr_D$ on the neutral stability curves for the fixed values of other governing parameters. From this figure it is evident that for small and moderate values of $\Pr_D$ the critical value of oscillatory Rayleigh number decreases with an increase of $\Pr_D$, this trend is reversed for large values of $\Pr_D$.

The effect of Lewis number Le on critical Rayleigh number is shown in Figs. 7(a) & (b). We observe that increase of Lewis number Le increases the critical Rayleigh number indicating that the Lewis number stabilizes the system in both stationary and oscillatory mode.

Figs. 8(a) & (b) depict the effect of solute Rayleigh number Ra_S on the neutral stability curves for stationary and oscillatory modes. We find that the effect of increasing Ra_S is to increase the value of the Rayleigh number for stationary mode.

The variation of Ra_{Tc} with Ra_S for different values of the internal Rayleigh number Ri is depicted in Fig. 9. We found that with an increase of Ri the critical Rayleigh number Ra_{Tc} decrease for both stationary and oscillatory modes. Thus the effect of Ri is to destabilize the system for both stationary and oscillatory modes.

Figure 10 displays the variation of Ra_{Tc} with Ra_S for different values of Taylor number Ta. It is observed that with an increase of Ta the oscillatory and the stationary critical Rayleigh number increase, implying that Ta has a stabilizing effect on both the system.

Figure 11 indicates the variation of Ra_{Tc} with Ra_S for different values of mechanical anisotropy parameter ξ for the fixed values of other parameters. It is important to note that Ra_{Tc} increases with an increase of mechanical anisotropy parameter ξ indicating its stabilizing characteristic.

Figure 12 shows the variation of Ra_{Tc} with Ra_S for different values of thermal anisotropy parameter η. It is observed that when $\eta > 1$, the critical Rayleigh number Ra_{Tc} increases with an increase of η indicating that the effect of increasing thermal anisotropy parameter is to delay the onset of stationary and oscillatory convection as compared to the isotropic case. On the other hand, when $\eta < 1$, the critical Rayleigh number Ra_{Tc} decreases with decrease of η indicating that the effect of decreasing thermal anisotropy parameter is to advance the onset of stationary and oscillatory convection as compared to the isotropic case.

The variation of Ra_{Tc} with Ra_S for different values of the Darcy-Prandlt number $\Pr_D$ is presented in Fig 13. From this figure it is found that the critical oscillatory Rayleigh number decreases with an increase of $\Pr_D$, for smaller values of Ra_S this trend reverses. Thus $\Pr_D$ has a dual effect on the system in oscillatory mode. Figure 14 shows the effect of normalized porosity parameter ϕ on the critical oscillatory Rayleigh number, when all other parameters are kept fixed. It is noticed that Ra_{Tc} decreases with an increase of normalized porosity parameter ϕ.

To understand the transient behavior the variation of Nusselt and Sherwood numbers with time has been considered and is depicted in Figs. 15(a) to (h) for different values of Ri, Ta, ξ *and* η. It is observed that both *Nu* and *Sh* start with a conduction state value close to 3 for $t > 0$. This periodic variation of *Nu* and *Sh* is very short lived and decays as time progresses. In other words, as time progresses a steady state is reached via a transient state.

From Figs. 15(a) & (b) it is clear that the heat transport increases slightly with increasing the internal Rayleigh number Ri and has insignificant influence on mass transport. Figs. 15(c) & (d) show the effect of the Taylor number Ta on heat and mass transport. We find that an increase in Ta, increase both Nu and Sh marginally. Figs. 15(d) & (e) we find that an increase in the value of mechanical anisotropic parameter ξ the heat transport increases slightly, while enhances the amplitude of the mass transport. The effect of thermal anisotropic parameter η is to decrease the amplitude of the oscillation of heat flux while it has no significant effect on mass flux and is shown in Figs. 15(g) & (h).

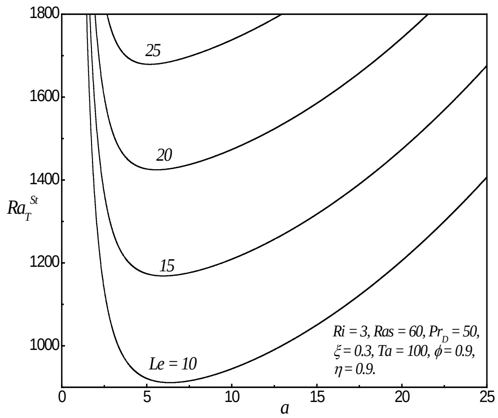


Fig 7(a). Stationary neutral stability curves for different values of Lewis number *Le*.

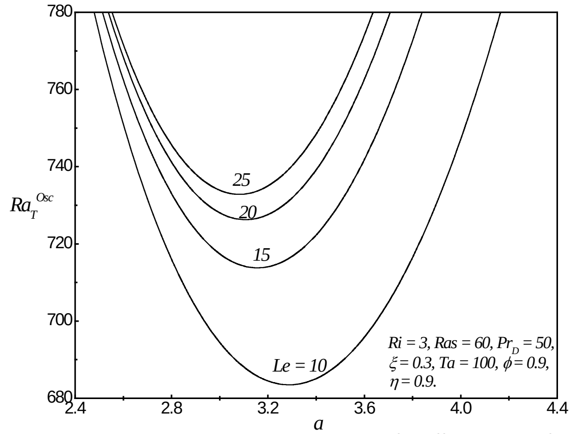


Fig 7(b). Oscillatory neutral stability curves for different values of Lewis number *Le*.

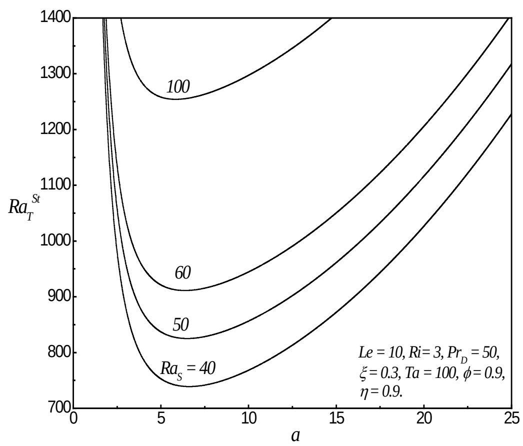


Fig 8(a). Stationary neutral stability curves for different values of Solute Rayleigh number Ra_S.

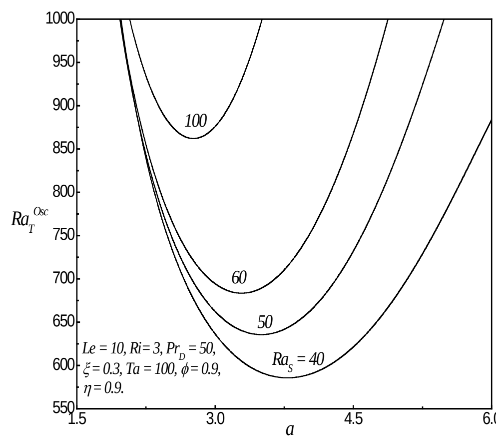


Fig 8(b). Oscillatory neutral stability curves for different values of Solute Rayleigh number Ra_S.

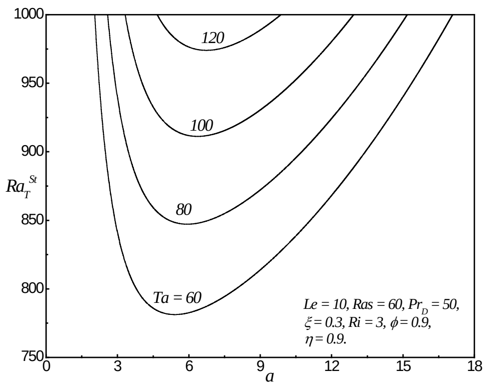


Fig 2(a). Stationary neutral stability curves for different values of Taylor number *Ta.*

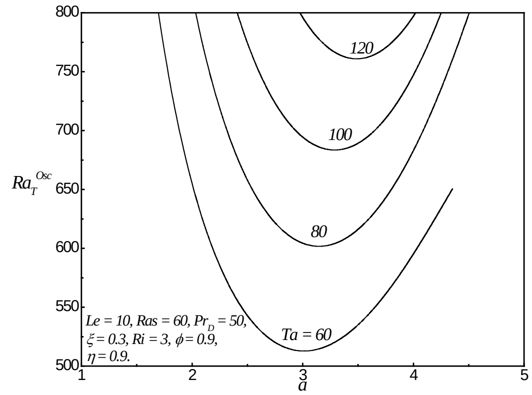


Fig 2(b). Oscillatory neutral stability curves for different values of Taylor number *Ta.*

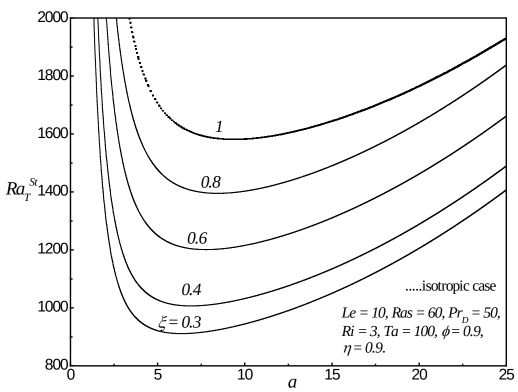


Fig 3(a). Stationary neutral stability curves for different values of Mechanical anisotropy parameter ξ.

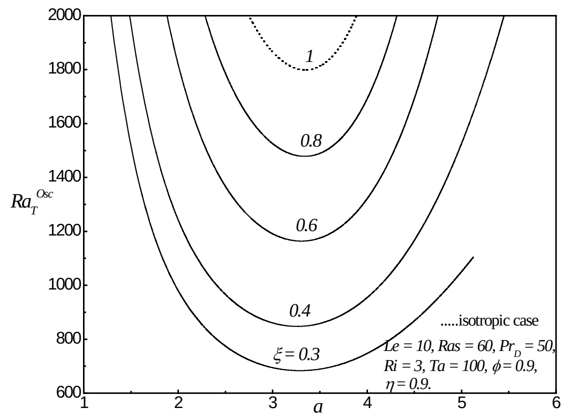


Fig 3(b). Oscillatory neutral stability curves for different values of Mechanical anisotropy parameter ξ

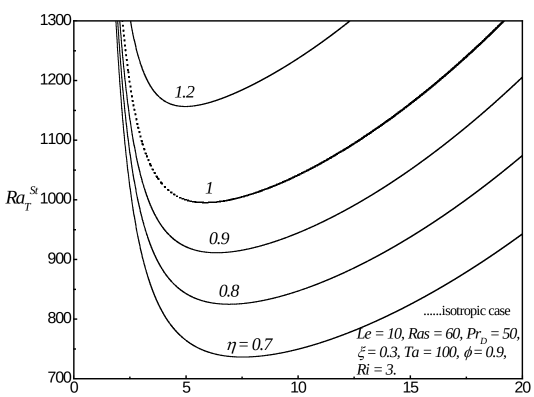


Fig 4(a). Stationary neutral stability curves for different values of Thermal anisotropy parameter η.

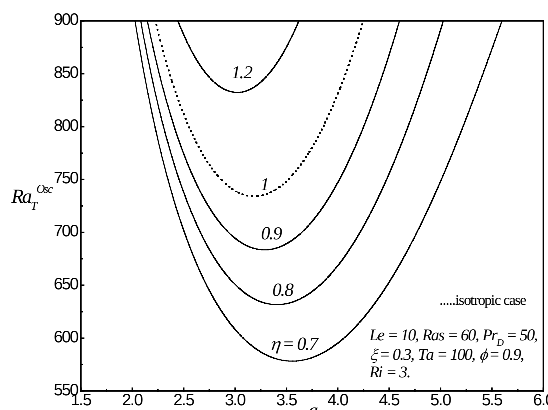


Fig 4(b). Oscillatory neutral stability curves for different values of Thermal anisotropy parameter η.

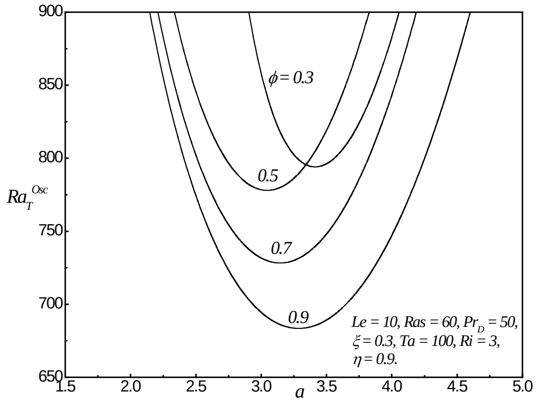


Fig 5. Oscillatory neutral stability curves for different values of Normalized porosity parameter ϕ.

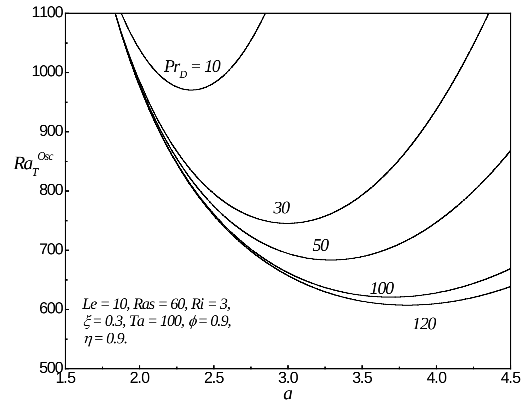


Fig 6. Oscillatory neutral stability curves for different values of Darcy-Prandlt number Pr_D.

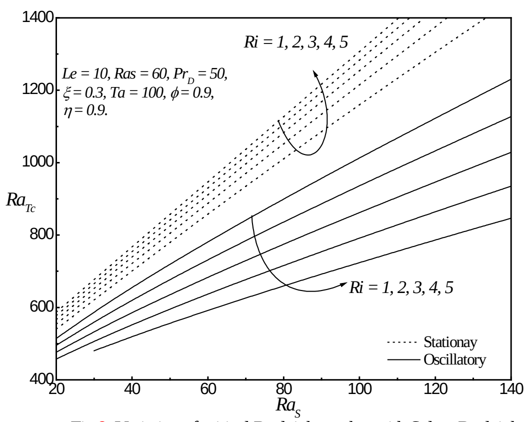


Fig 9. Variation of critical Rayleigh number with Solute Rayleigh number for different values of an internal Rayleigh number *Ri*.

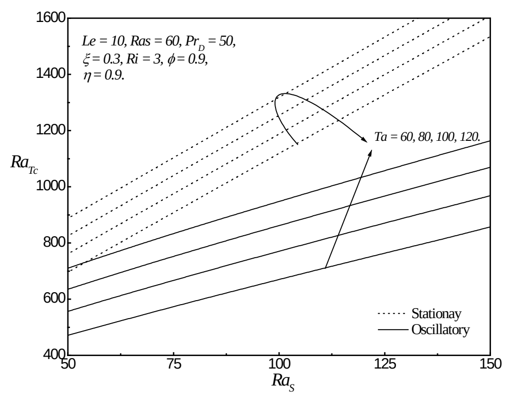


Fig 10. Variation of critical Rayleigh number with Solute Rayleigh number for different values of Taylor Number *Ta*.

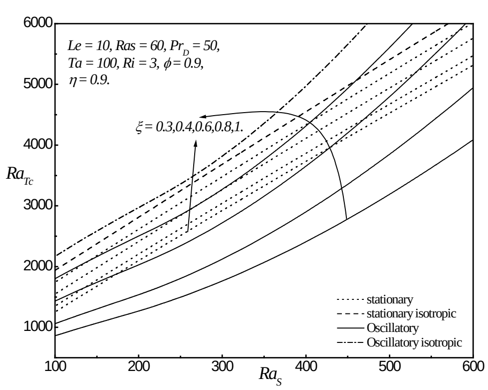


Fig 11. Variation of critical Rayleigh number with Solute Rayleigh number for different values of Mechanical anisotropy parameter ξ

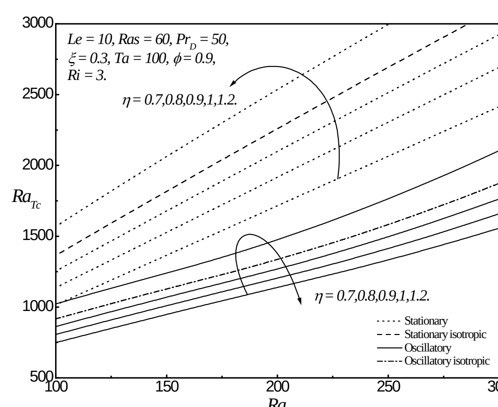


Fig 12. Variation of critical Rayleigh number with Solute Rayleigh number for different values of Thermal anisotropic parameter η.

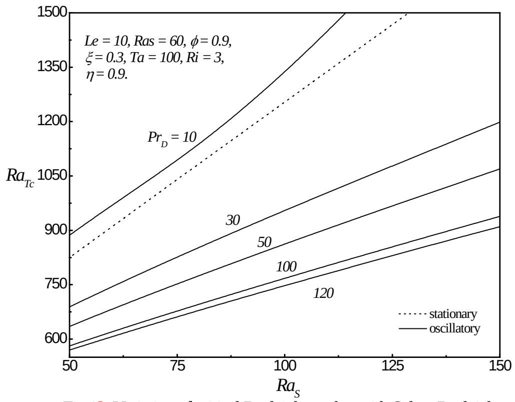


Fig 13. Variation of critical Rayleigh number with Solute Rayleigh number for different values of Darcy-Prandlt number Pr_D.

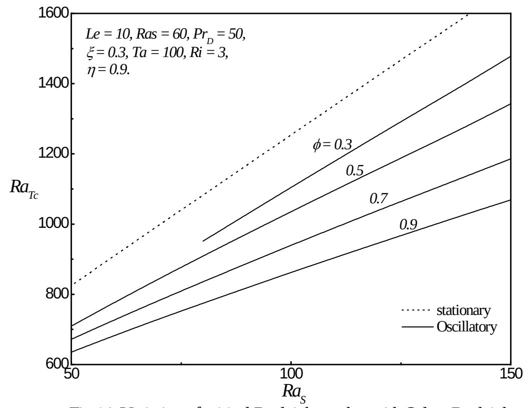
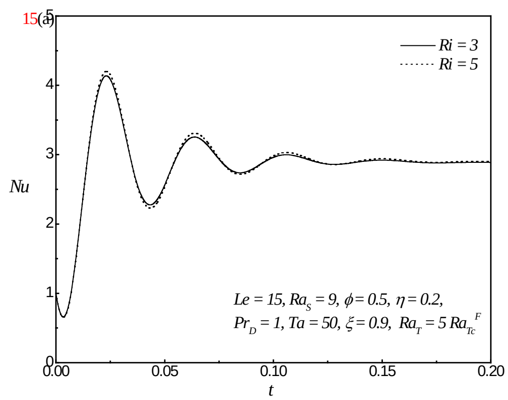
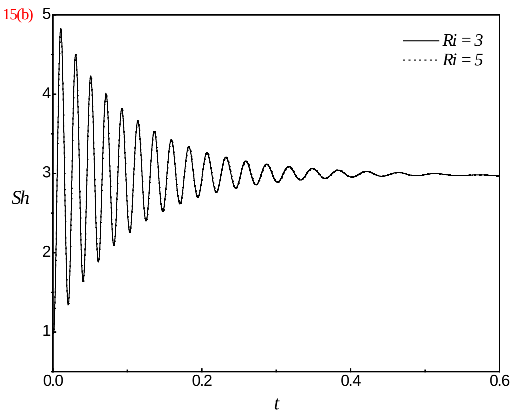


Fig 14. Variation of critical Rayleigh number with Solute Rayleigh number for different values of Normalized porosity parameter ϕ.

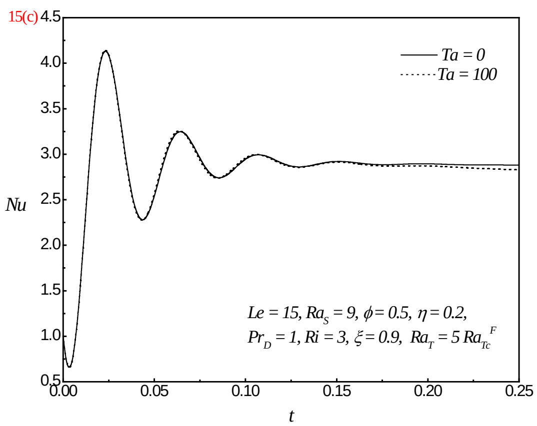
15(a)



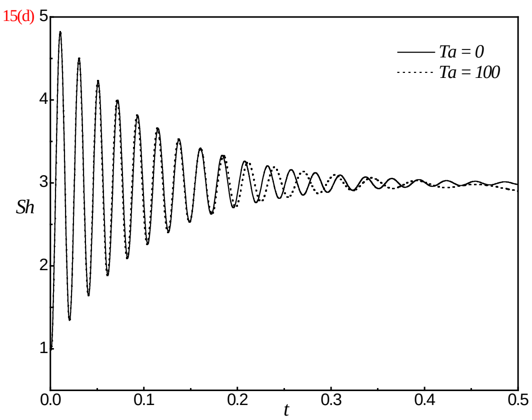
15(b)

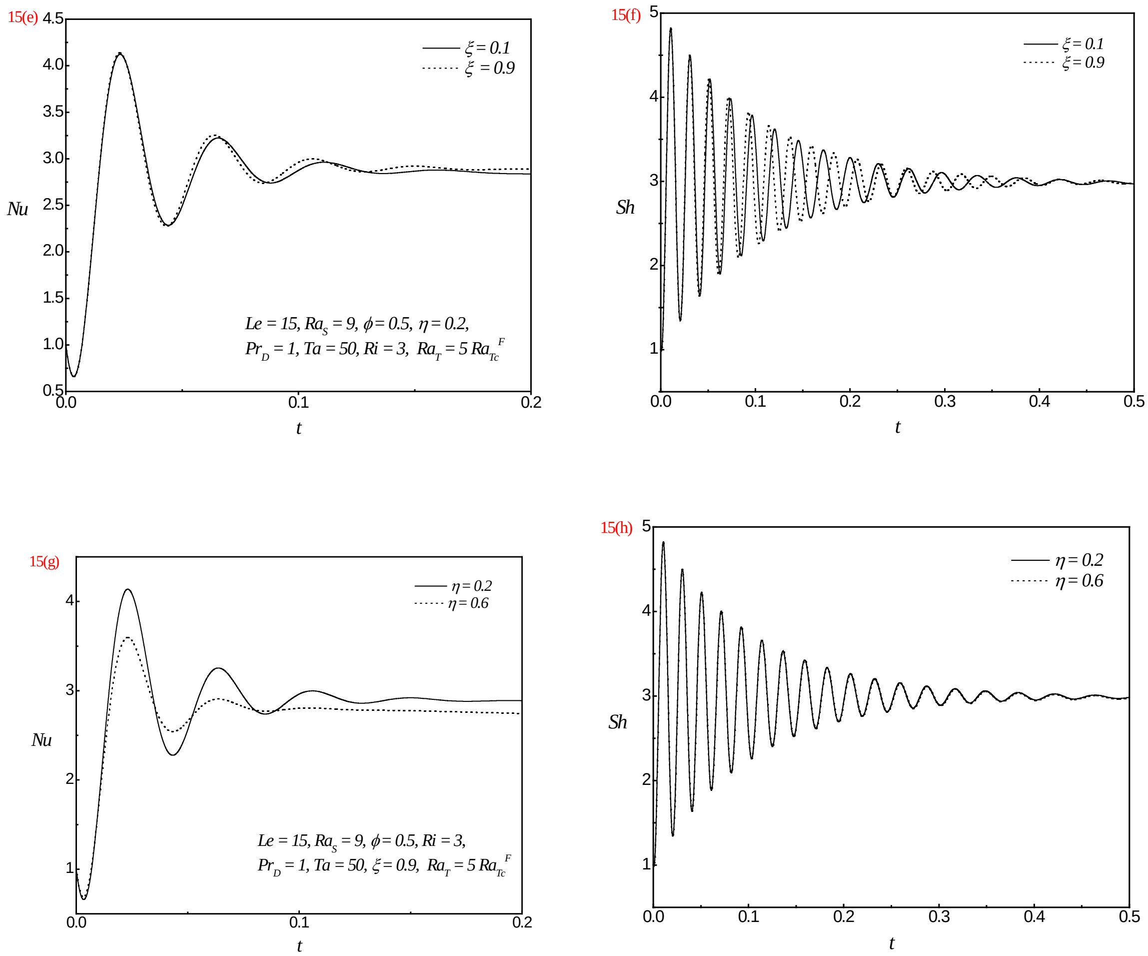


15(c)



15(d)





Figs. 15 (a) & (b) Variation of *Nu* and *Sh* with time for different values of Ri ; (c) & (d) Variation of *Nu* and *Sh* with time for different values of Ta ; (e) & (f) Variation of *Nu* and *Sh* with time for different values of ξ ; (g) & (h)Variation of *Nu* and *Sh* with time for different values of η .

CONCLUSION

The effect of rotation on the onset of double diffusive convention in a saturated anisotropic porous layer with internal heat source is investigated analytically using both linear and weak nonlinear stability analysis. The linear theory provides the onset criteria for both stationary and oscillatory convection. The nonlinear theory which is based on the truncated Fourier technique provides a mean to measure the convection amplitudes and the rate of heat and mass transfer. The main conclusions of the present study are as follows.

- From the neutral stability curves, it is found that the effects of increasing Ri, ϕ *and* $\Pr_D$ are to advance the onset of convection, whereas the effect of increasing Ta, ξ, η, Le *and* Ra_S is to sustain the stability of the system.
- Critical value of Ra_T decreases as the values of $Ri, \Pr_D$ *and* ϕ increase, while it increases with increasing Ta, ξ *and* η .

- Heat transfer increases slightly with an increase of $Ri, Ta\ and\ \xi$ and decreases with an increase of η.
- Mass transfer increases with increase of $Ta\ and\ \xi$ and has no significant effect on $Ri\ and\ \eta$.

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Nomenclature

a	:	Wave number $\sqrt{l^2+m^2}$
d	:	Height of the porous layer
$\mathbf{g}$	:	Gravitational acceleration $(0,0,-g)$
H	:	rate of heat transport per unit area
$\mathbf{K}$	:	permeability tensor, $K_x^{-1}(\mathbf{ii}+\mathbf{jj})+K_z^{-1}(\mathbf{kk})$
Le	:	Lewis number, κ_T/κ_S
l,m	:	Horizontal wavenumbers
Nu	:	Nusselt number
P	:	Pressure
$\Pr_D$	:	Darcy–Prandtl number, $\gamma\varepsilon\nu d^2/K_z\kappa_{Tz}$
$\mathbf{q}$	:	Velocity vector, (u,v,w)
Ra_S	:	Solute Rayleigh number, $\beta_S g\Delta T d K_{Tz}/\nu\kappa_{Tz}$

Ra_T	:	Thermal Rayleigh number, $\beta_T g \Delta T d K_{Tz} / \nu \kappa_{Tz}$
Ri	:	Internal Rayleigh number, Qd^2 / κ_{Tz}
S	:	Solute concentration
Sh	:	Sherwood number
ΔS	:	Salinity difference between the walls
Ta	:	Taylor number, $\left(2\Omega K_z / \varepsilon \nu\right)^2$
t	:	Time
T	:	Temperature
ΔT	:	Temperature difference between the walls
x, y, z	:	Space coordinates

Greek Symbols

β_T	:	Thermal expansion coefficient
β_S	:	Solute expansion coefficient
Φ	:	Dimensionless amplitude of concentration perturbation
ϕ	:	Normalized porosity, ε / γ
γ	:	Ratio of specific heats, $\left(\rho c\right)_m / \left(\rho c_p\right)_f$
η	:	Thermal anisotropy parameter, $\kappa_{Tx} / \kappa_{Tz}$
κ_T	:	Anisotropic thermal diffusion tensor
κ_S	:	Solute diffusivity
κ	:	Diffusivity
ε	:	Porosity
μ	:	Dynamic viscosity
μ_e	:	Effective viscosity
ν	:	Kinematic viscosity, μ / ρ_0
Θ	:	Dimensionless amplitude of temperature perturbation
ρ	:	Fluid density
σ	:	Growth rate
$\boldsymbol{\Omega}$	:	Angular velocity of rotation, $\left(0, 0, \Omega\right)$
ξ	:	Mechanical anisotropy parameter, K_x / K_z
ψ	:	Stream function

Other Symbols

∇_h^2	$\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$
∇^2	$\nabla_h^2 + \dfrac{\partial^2}{\partial z^2}$

Subscripts & Superscripts

b	:	Basic state
c	:	Critical

f	:	Fluid
h	:	Horizontal
m	:	Porous medium
0	:	Reference value
S	:	Solid
*	:	Dimensionless quantity
'	:	Perturbed quantity
F	:	Finite amplitude
Osc	:	Oscillatory state
St	:	Stationary state

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