

Journal of Chemical, Biological and Physical Sciences



An International Peer Review E-3 Journal of Sciences

Available online at www.jcbpsc.org

Section C: Physical Sciences

CODEN (USA): JCBPAT

Research Article

A theoretical study of strongly coupled plasma and an evaluation of electrical conductivity of strongly coupled tungsten plasma

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Received: 19 March 2015; **Revised:** 22 March 2015 **Accepted:** 25 March 2015

Abstract: We have studied the physical behavior of strongly coupled plasma and theoretically analyzed the experimental data of electrical conductivity of strongly coupled tungsten plasma. We have studied the influence of equation of state (EOS) model in the interpretation of electrical conductivity of tungsten plasma. The theoretical study has been performed by the theoretical formalism of D. A. Baiko, Phys. Rev E80 (2009)⁸. Our theoretical results show that the results obtained from EOS3 is quite near to the experimental data.

Keywords: Strongly coupled plasma, one-component plasma (OCP), Plasma in controlled thermonuclear experiments, highly evolved stars, degenerate plasma, Plasma dynamics, Plasma transport coefficient, equation of states, Numerical simulation

INTRODUCTION

The plasma is a statistical system of mobile charged particles. The charge particles interact with each other via the electromagnetic forces. To answer what a strongly coupled plasma is, one might begin with a specification of the strong coupling concept in the plasma. For simplicity, one considers spatially homogeneous one-component plasma (OCP). It is a system consisting of a single species of charged particles embedded in a uniform background of neutralizing charges. Such an OCP is a substantially idealized model for real plasmas.

One defines the coupling constant of the plasma as the ratio of the average Coulomb energy to the average kinetic energy. One is concerned with the strength of coupling due to coulomb interaction. Those plasmas with values of the coupling constant $\Gamma > 1$ may be called strongly coupled plasma. Most of the classical plasmas are characterized by $\Gamma \ll 1$. For example, one may assume $n = 10^{11} \text{cm}^{-3}$ (electron density), $T = 10^4 \text{K}$ for gaseous discharge plasma in laboratory, $n = 10^{16} \text{cm}^{-3}$, $T = 10^8 \text{K}$ for a plasma in a controlled thermonuclear experiment, $n = 10^{16} \text{cm}^{-3}$, $T = 10^6 \text{K}$ for a plasma in the solar corona. For those plasmas, one finds $\Gamma = 10^{-3}$, 10^{-5} and 10^{-7} respectively. These are weakly coupled plasmas and their thermodynamic properties are analogous to those of an ideal gas.

A most typical example of strongly coupled classical plasma may be seen in the system of ions inside a highly evolved star^{1,2}. The interior of such a star is in compressed and high density state. The Fermi energy of the electron system takes on a value much greater than the binding energy of an electron around an atomic nucleus, all the atoms are thus in ionized state. The electron system constitutes a weakly coupled ($r_s \gg 1$) degenerate plasma with an immensely large Fermi energy ($E_F = mc^2$). It makes an ideal neutralizing background of negative charges to the ion system; since $r_s \gg 1$, the polarizability of the background may be negligible. Those atomic nuclei stripped of the electrons form an ion plasma obeying the classical statistics, their de Broglie wavelengths are much smaller on the average than the inter particle spacing. In the interior of an evolved star, the coupling constant Γ for such an ion plasma is greater than unity. In case of White dwarf star, one estimates that $\Gamma = 10$ -200.

Strongly coupled plasma is used commonly in fundamental research and high technology applications³. For a prediction of plasma dynamics, it is necessary to know its thermo physical properties depending on temperature T and density ρ . There are a scant set of experimental data on plasma properties in the range of $T = 1$ -5 eV density $\rho = 1 \text{g/cm}^3$ so far. A reliable theoretical model of plasma transport coefficients in this range of parameters does not exist also. Therefore an investigation of equation of state (EOS) and transport properties of plasmas in the mentioned range of parameters is very interesting.

Optical opacity of plasma impoverishes significantly diagnostic techniques of experiments. In this case a volumetric homogeneity of a studying sample is a principle requirement for specific quantities determination. A new method of measurements of the electrical resistivity (conductivity) of plasma based on explosion of metal foils under high-power current pulse was proposed⁴. In the present chapter, one studies the influence of EOS model on the interpretation of data from experiment⁵ for tungsten.

The measurements were carried out in a plane geometry. A tungsten foil stripe with length $l_z = 10 \text{mm}$, width $h = 1.5 \text{mm}$ and thickness $2a = 5 \text{mm}$. Side slits were shielded with thin mica stripes. In the experiment under consideration the skin layer thickness δ is significantly larger than the foil thickness. Cartesian coordinate system is introduced as follows: x-axis is perpendicular to the foil plate, y-axis is directed along the smaller side of the foil and z-axis is along the bigger side. In 1D process the foil expands along

the x-axis, the magnetic induction B is directed along the y-axis and the heating current I as well as the electric field intensity E are directed along the z-axis.

The foil was heated by the current pulse, the time dependence of the current through the sample $I(t)$ and voltage drop $U(t)$ were registered. Then resistive part of the voltage drop $U_R(t)$ was calculated. Electrical resistance $R(t)=U_R(t)I^{-1}(t)$ and Joule heat $q(t)$ were calculated. Other values required for conductivity calculation can be obtained with the help of numerical simulation. Assuming that the current density j is distributed uniformly over the cross-section of the foil and depends only in time i.e. $j(t)=I(t)S^{-1}(t)$ where

$S(t)=2a(t)h$ from Maxwell equation $j(t)=\mu^{-1}\frac{\partial B}{\partial x}$ (SI system of units are used, μ is magnetic permeability)

one can calculate $B(t,x)=\mu I(t)xS^{-1}(t)$. So it is possible to determine the x-t dependences of foil parameters as a numerical solution of only a set of hydrodynamic equations with the Ampere force $jB=\mu I^2(t)xS^{-2}(t)$ and energy input $jE=U(t)I(t)V^{-1}(t)$ where $V(t)=S(t)l_z$ is the foil volume, h is width.

The results of calculation by such a technique not showing for magnetic diffusion were presented in this work.

Mathematical formula used in the evaluation: One assumes that spatial perturbations of the sample are small. Electron and ion temperatures are same. One neglects the thermal conductivity effect. One uses the set of 1D magneto hydrodynamic (MHD)⁶ equations in Lagrangian form for the foil heating. The equations are following:

$$\frac{dm}{dt} = 0 \quad (1)$$

$$\rho \frac{dv}{dt} = -\frac{\partial P}{\partial x} - (2\mu)^{-1} \frac{\partial B^2}{\partial x^2} \quad (2)$$

$$\rho \frac{d\varepsilon}{dt} = -P \frac{\partial v}{\partial x} + \partial \left(\frac{\kappa \partial T}{\partial x} \right) / \partial x + \frac{j^2}{\sigma} \quad (3)$$

$$d(\mu B) / dt = \partial(\sigma^{-1} \partial B / \partial x) / \partial x \quad (4)$$

Where m is the mass, v is the particle velocity, ρ is the density, T is the temperature, P is the pressure, ε is the specific internal energy, σ is the electrical conductivity and κ is the thermal conductivity. Initial conditions for the set of above equations (1)-(4) are written as follows

$$\begin{aligned} \rho(x, 0) &= \rho_0 \\ v(x, 0) &= 0 \\ T(x, 0) &= T_0 \\ B(x, 0) &= 0 \end{aligned} \quad (5)$$

The other conditions on symmetry plane $x=0$ and the surface $x=a(t)$ of the foil as well as on the outer boundary of the glass plate $x=a_1$ are as follows

$$v(0,t) = 0$$

$$v(a,t) = \frac{da}{dt}$$

$$v(a_1,t) = 0$$

$$B(0,t) = 0$$

$$B(a,t) = -\mu I(t)/2h$$

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = 0 \quad (6)$$

The other boundary conditions are

$$\left(\frac{\partial T}{\partial x}\right)_{x=a-0} = \left(\frac{\partial T}{\partial x}\right)_{x=a+0}$$

$$T(a_1,t) = T_0$$

$$\left(\frac{\partial P}{\partial x}\right)_{x=0} = 0$$

$$P(a-0,t) = P(a+0,t)$$

$$P(a_1,t) = P_0 \quad (7)$$

Here ρ_0 , T_0 and P_0 correspond to normal conditions. The electrical resistance is given by

$$R(t) = U_R(t) I^{-1}(t) \quad (8)$$

Where $I(t)$ is current and $U_R(t)$ is Voltage drop

$$V(t) = S(t) l_z \quad (9)$$

The electrical conductivity is given by

$$\sigma = I(t) l_z U^{-1}(t) S^{-1}(t) \quad (10)$$

RESULTS

In this paper, we have studied the physical behavior of strongly coupled plasma. We have theoretically evaluated the electrical conductivity of strongly coupled tungsten plasma. Electrical explosion of wire or foils is an effective way to study thermo physical properties of matter in a wide range of densities and temperature⁷. Here, we have studied the influence of equation of state (EOS) model in the interpretation of electrical conductivity measurements in strongly coupled tungsten plasma. The experimental data was taken from the work of Korobenka *etal*⁵. **In table 1**, we have shown the numerical simulation results of current $I(\text{kA})$ and voltage (kV) as a function of $t(\mu\text{s})$ from measured data of Korobenka *etal*⁵. The numerical simulation was performed with conductivity model taking into account breakdown effects as well as disregarding breakdown.⁸ **In table 2**, we have shown the evaluated results of specific internal

energy ϵ (KJ/g) as a function of time (μ s) in the foil during heating. Calculations have been performed for simulations using three equations of state E01, E02 and E03. From the table, it is quite evident that the value of $\epsilon(t)$ are very close to each other during the initial heating stage. **In table 3**, we have shown the evaluated results of Pressure as a function of specific internal energy in the foil during the heating. Our Theoretical results shows that the calculated pressure in the process of foil heating is always lower than the present work. All the three equations of states (E01, E02 and E03) gives identical results. **In table 4**, we have shown the evaluated results of specific electrical resistivity of tungsten as a function of specific internal energy in the foil during the heating. Calculations were performed by all the three equations of state and results were compared with the experimental data. It appears that calculation performed by E03 is quite near to the experimental results. There is some recent calculation⁹⁻¹⁵ on strongly coupled plasma which also reveals the similar facts.

Table 1: A numerical simulation results of current I (KA) and voltage V(KV) as a function of t (μ s) from measured data of Korobenko *et al.* The numerical simulation were performed with conductivity model taking into account breakdown effect as well as disregarding breakdown

t (μ s)	I(KA)	V(KV)
0.1	7.85	2.37
0.2	8.15	3.49
0.3	9.23	4.56
0.4	10.42	5.78
0.5	11.37	6.29
0.6	12.48	7.86
0.7	13.57	8.20
0.8	11.28	9.58
0.9	10.86	8.45
1.0	12.59	7.53
1.1	14.53	6.44
1.2	16.78	5.56

Table 2: An evaluated results of specific internal energy ϵ (KJ/g) as a function of t (μ s) in the foil during heating. Calculations were performed using three equations of states (E01, E02 and E03) and results were compared with the experimental data

t (μ s)	ϵ (KJ/g)			
	E01	E02	E03	Expt data
0.1	0.025	0.036	0.042	0.030
0.2	0.058	0.097	0.088	-----
0.3	0.357	0.386	0.407	0.535
0.4	1.567	1.558	1.549	-----
0.45	1.672	1.653	1.646	-----
0.50	1.987	1.954	1.932	-----
0.55	2.168	2.146	2.139	-----
0.60	3.846	3.792	3.804	2.458
0.65	4.108	4.087	4.005	-----
0.70	5.349	5.167	5.058	4.155

Table 3: An evaluated result of pressure as a function of specific internal energy in the foil during heating, Calculations were performed by all the three equations of state and results were compared with experimental data.

$\epsilon(\text{KJ/g})$	$P(\text{GPa})$			
	E01	E02	E03	Expt data
0.5	0.582	0.652	0.642	0.603
1.0	0.684	0.784	0.765	0.754
2.0	0.873	0.895	0.886	0.852
3.0	0.987	1.087	1.067	1.054
4.0	1.128	1.143	1.138	1.122
6.0	1.562	1.674	1.654	1.638
8.0	3.674	3.786	3.685	3.645
9.0	3.832	3.875	3.854	3.825
10.0	4.165	4.207	4.172	4.106
11.0	4.386	4.415	4.374	4.254
12.0	4.532	4.567	4.505	4.446

Table 4: An evaluated result of specific electrical resistivity of tungsten as a function of specific internal energy in the foil during heating Calculations were performed by all the three equations of state and results were compared with the expt.data.

$\epsilon(\text{KJ/g})$	$\sigma^{-1}, \mu\Omega\text{m}$			
	E01	E02	E03	Expt. data
0.5	2.652	3.154	3.116	3.102
1.0	3.187	3.482	3.584	3.476
2.0	4.527	4.643	4.163	4.115
3.0	5.287	5.334	5.268	5.252
4.0	5.654	5.706	5.654	5.384
5.0	6.154	6.185	6.125	6.087
6.0	6.784	6.786	6.728	6.689
8.0	8.342	7.356	7.198	7.055
10.0	15.654	14.512	13.869	13.632
11.0	16.253	15.674	14.532	14.604
12.0	18.547	17.258	17.186	16.980

CONCLUSION

In this paper, we have analyzed the experimental data of electrical conductivity of strongly coupled tungsten plasma under heating by current probe.

- (1) Here, we have used 1D MHD simulation and different EOS models to study distribution of parameters in the foil. We have tried to reproduce experimental voltage time dependence using two electrical conductivity models.
- (2) Our theoretical studies indicate that the pressure, density and temperature are distributed almost homogeneously across the foil except at the melting stage of the process.

- (3) The dynamics of the heating and expansion is determined by the equation of state EOs model and EOS model3 works quite well in reproducing the experimental results for σ^{-1} .

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