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LRS Bianchi Type V Perfect Fluid Cosmological Model in C – Field Theory with Variable Λ

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Abstract: A cosmological model for perfect fluid distribution in creation field cosmology with varying cosmological constant (Λ) in Bianchi type - V space- time is investigated. To get deterministic model satisfying conservation equation, we assume $\Lambda = 1/R^2$ as considered by Chen and Wu[1] where R is scale factor. We find that creation field (C) increase with time, $\Lambda \sim 1/t^2$, which matches with the result of HN theory. The decelaration parameter $q < 0$ which shows that universe is accelarating. The model is free from horizon.

Key Words: LRS Bianchi type V, Perfect fluid, C- field, Variable Λ.

INTRODUCTION

The phenomenon of expanding universe, primordial nucleosynthesis and the observed isotropy of cosmic microwave background radiation (CMBR) are the three very important observations in astronomy. These were successfully explained by big-bang cosmology based on Einstein's field equations. Smoot et.al.[2] revealed that the earlier predictions of the Friedman-Robertson Walker type of models do not always exactly match with our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue to contradict the theoretical explanations given from the big-bang type of the model. Also, CMBR discovery did not prove it to be a outcome of big-bang theory. In fact, Narlikar et. al.[3] have proved the possibility of non-relic interpretation of CMBR. To explain such phenomenon, many alternative theories have been proposed from time to time. Hoyle[4], Bondi and

Gold[5] proposed steady state theory in which the universe does not have singular beginning nor an end on the cosmic time scale. To overcome this difficulty Hoyle and Narlikar[6] adopted a field theoretic approach by introducing a massless and chargeless scalar field C in the Einstein-Hilbert action to account for the matter creation. In the C-field theory, introduced by Hoyle and Narlikar there is no big-bag type of singularity as in the steady state theory of Bondi and Gold[5]. A solution of Einstein's field equations admitting radiation with negative energy massless scalar creation field C was obtained by Narlikar and Padmanabhan[1].The study of Hoyle and Narlikar theory[6–8] to the space-times with higher dimensions was carried out by Chatterjee and Banerjee[9]. Bali and Tikekar[10] studied C-field cosmology with variable G in the flat Friedmann-Robertson-Walker model and with non-flat FRW space-time by Bali and Kumawat[22]. The solutions of Einstein's field equations in the presence of creation field have been obtained for different Bianchi type universes by Singh and Chaubey[11].

The non-trivial role of vacuum generates a cosmological constant (Λ) term in Einstein's field equations which leads to the inflationary scenario (Abers and Lee[12]) which predicted that during an early exponential phase, the vacuum energy is treated as large cosmological constant which is expected by Glashow-Salam-Weinberg and by Grand Unified Theory as mentioned by Langacker[13]. Therefore, the present day observations of smallness of cosmological constant $\Lambda \leq 10^{-56}\text{cm}^{-2}$ support to assume that cosmological constant is time dependent. Gibbons and Hawking[14] investigated that cosmological models with positive cosmological constant leads to de-Sitter space-time asymptotically. Therefore, the cosmological models linking the variation of cosmological constant having the form of Einstein's field equations unchanged and preserving the energy-momentum tensor of matter content, have been studied by several authors viz. Berman[15], Abdussattar and Vishwakarma[16], Bali and Singh[17], Pradhan et al.[18], Bali and Jain[19], , Bali and Tinker[20], Ram and Verma[21].

Motivated by aforesaid, in this letter we have investigated cosmological model for perfect fluid distribution in C- field cosmology with varying cosmological constant in LRS Bianchi type-Vspace-time. For deterministic model,we assumed $\Lambda = 1/R^2$, where R is scale factor. We find that creation field (C) increase with time, $\Lambda \sim 1/t^2$. Physical and geometrical aspect of the model are also discussed. These model is free from horizon.

THE METRIC AND FIELD EQUATION

We consider homogenous LRS Bianchi type-V metric in the form of

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}(dy^2 + dz^2) \qquad (1)$$

Where A and B are functions of t alone.

The Einstein's field equation (C = 1)by introduction of C-field is modified by Hoyle- Narlikar [3] with time dependent cosmological term is given by

$$R_j^i - \frac{1}{2} R\, g_j^i = -8\pi G\left[(m)T_j^i + (c)T_j^i\right] - \Lambda\, g_j^i \qquad (2)$$

The energy-momentum tensor ${}_{(m)}T_j^i$ for perfect fluid and creation field ${}_{(c)}T_j^i$ are given by

$${}_{(m)}T_j^i = (\rho + p)v_jv^i - p\, g_j^i \qquad (3)$$

$$_{(c)}T_j^i = -f\left(c_j\, c^i - \frac{1}{2} g_j^i c_\alpha c^\alpha\right) \tag{4}$$

Where f > 0 is coupling constant between the matter and creation field and $C_i = \frac{dc}{dx^i}$.

The coordinate are chosen to co-moving such that $v^i = (0, 0, 0, 1)$.

Which gives

$$T_1^1 = -p = T_2^2 = T_3^3\,,\ T_4^4 = \rho \text{ for matter.} \tag{5}$$

$$T_1^1 = \frac{1}{2} f\dot{c}^2 = T_2^2 = T_3^3\,, T_4^4 = -\frac{1}{2} f\dot{c}^2 \qquad \text{for C – field.} \tag{6}$$

Thus, $$T_1^1 = \left(-p + \frac{1}{2} f\dot{c}^2\right) = T_2^2 = T_3^3, T_4^4 = \left(\rho - \frac{1}{2} f\dot{c}^2\right) \tag{7}$$

Hence the Einstein field equation (2) for the metric (1) and EMT (3), (4) takes the form

$$2\,\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{A^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{9}$$

$$2\,\frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{3}{A^2} = 8\pi G\left(\rho - \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{10}$$

$$2\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0 \tag{11}$$

The Suffix 4 by the symbols A and B denotes, a differentiation with respect to *t*.

SOLUTION OF FIELD EQUATIONS:

The conservation equation of EMT

$$\left(8\pi G T_j^i + \Lambda\, g_j^i\right)_{;i} = 0 \tag{12}$$

Which leads to

$$8\pi G\left[\dot{\rho} - f\dot{c}\ddot{c} + \{(\rho + p) - \ddot{}\, f\dot{c}^2\}\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\right] + \dot{\Lambda} = 0 \tag{13}$$

Following Hoyle and Narlikar [3] , the source equation of C-field : $c_{;i}^{\,i} = n/f$ leads to c = t for large r. thus $\dot{c} = 1$.

Equation (11) leads to

$$A = l\, B \tag{14}$$

Without loss of generality, we assume $l = 1$, using equation (14) into (8) - (10) lead to

$$2\,\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{B^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{15}$$

$$3\,\frac{B_4^2}{B^2} - \frac{3}{B^2} = 8\pi G\left(\rho - \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{16}$$

Following Hoyle and Narlikar[6] we assume zero pressure in matter field i.e. p = 0 and $\dot{c} = 1$.

Equation (15) leads to

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{B^2} = 4\pi Gf + \Lambda \qquad (17)$$

For the deterministic solution of eq. (17) we assume

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} \qquad (18)$$

From equation (17) and (18) together leads to

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} = 4\pi Gf + \frac{2}{B^2} \qquad (19)$$

Equation (19) leads to

$$2B_{44} + \frac{B_4^2}{B} = KB + \frac{2}{B} \text{ where } K = 4\pi Gf \qquad (20)$$

Let $B_4 = f(B)$ which leads to $B_{44} = ff'$

Equation (20) leads to

$$\frac{df^2}{dB} + \frac{1}{B}f^2 = KB + \frac{2}{B} \qquad (21)$$

Equation (21) leads to

$$f^2 = \left(\frac{dB}{dt}\right)^2 = \alpha B^2 + 2, \text{ where } \alpha = \frac{K}{3} \qquad (22)$$

The constant of integration has been taken zero for simplicity.

Equation (22) leads to

$$\frac{dB}{\sqrt{B^2 + a^2}} = \beta dt \qquad (23)$$

Where $a^2 = \frac{6}{K}$ $and\ \beta = \sqrt{\frac{K}{3}}$

Equation (23) leads to

$$B = a \sinh \beta t \qquad (24)$$

Now $p = 0$ and eq. (14) together with eq. (13) leads to

$$8\pi G\left[\dot{\rho} - f\dot{c}\ddot{c} - f\ddot{\dot{c}}^2\left(3\frac{B_4}{B}\right) + \rho\left(3\frac{B_4}{B}\right)\right] + \dot{\Lambda} = 0 \qquad (25)$$

Equation (25) together with eq. (24),(31) and (32) leads to

$$\frac{d\dot{C}^2}{dt} + (6\coth t)\dot{C}^2 = (6\coth t)$$

$$where\ 4\pi fG = 3\ as\ \beta = 1 \qquad (26)$$

Equation (26) leads to

$$\dot{C}^2 = 1 \qquad (27)$$

Thus, we have

$$\dot{C} = 1 \qquad (28)$$

Which agrees with the value used in the source equation. Thus creation field is proportional to time t and the metric (1) for constraints mentioned above, leads to

$$ds^2 = dt^2 - (2\sinh^2 t)[dx^2 + e^{2x}(dy^2 + dz^2)] \qquad (29)$$

PHYSICAL AND GEOMETRICAL ASPECTS:

The homogeneous mass density ρ, the cosmological constant Λ, the scale factor R and deceleration parameter q for the model (29) are given by

$$R = B = \sqrt{2}\, sinh\, \beta t \tag{30}$$

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} = \frac{1}{2} cosech^2 \beta t \tag{31}$$

$$8\pi G\rho = cosech^2 t + 3 + K \tag{32}$$

$$q = -tanht \tag{33}$$

CONCLUSIONS

The scale factor R increase with time. The cosmological constant Λ decreases as time increases. Since the deceleration parameter q < 0, hence the model (29) represent an accelerating universe.Also the coordinate distance (γ_H) to the horizon is maximum distance a null ray could have travelled at time t starting from infinite past i.e.

$$\gamma_H(t) = \int_{-\infty}^{t} \frac{dt}{R(t)}$$

We could extent the proper time t to in the past because of the nonsingular nature of the space-time,

$$\gamma_H(t) = \int_0^t \frac{dt}{R(t)} = \int_0^t \frac{dt}{\sqrt{2} sinht}$$

The integral diverges at lower limit showing that the model (29) is free from particle horizon, thus creation field cosmology solves one of the outstanding problem (Horizon problem) faced by Big-Bang cosmology.

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