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Hybrid IDM Algorithm for PIV

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Abstract: The accuracy of the measurements using PIV method is largely dependent not only on the quality of the measured data, but also on the method of evaluation of seeding particle displacement. Our work deals with the modification of nowadays already standard algorithm based on mutual deformation of individual images and combines this algorithm with a common iterative scheme that uses mutual integer displacement of particular investigated areas. Our designed and tested algorithm enables the user to increase the accuracy of the measured data, while the computation time necessary for the analysis is not increased in order. The article describes in detail the procedure of designing and testing of the final algorithm. In order to test the suitability of particular algorithms, it was first necessary to generate synthetic data with a known displacement and of adequate quality. Apart from modifying the classical iterative algorithm we also focused on a method to detect the exact position of the signal peak with the help of regression. Individual partial changes in the algorithm were tested and mutually compared so that the best combination can be determined. Finally, the resultant algorithm was comprehensively tested.

Key words: Accuracy, algorithm, deformation, multistep, PIV.

INTRODUCTION

PIV (Particle Image Velocimetry) is a method used for measurement of velocity of fluids that allows obtaining information about stationary and non-stationary velocity field of single and multiphase systems. In the observed plane or volume, two or three velocity components can be measured. A significant advantage of the method is that, when used properly, it does not influence the flow and it is therefore a non-invasive method.

These characteristic features are the reason why the method has become a widely used tool when studying the phenomena in fluid mechanics. Since the use of the PIV method allows us to obtain information on the flow field in the whole measured plane at one time, the method is also used for verification and testing of numerical simulations[1]. When taking measurements using the PIV method, the ability of suitable "seeding" particles to follow the liquid flow is utilized. The principle of the PIV method is to determine displacement of particles within the observed volume in a known time interval. The measuring accuracy of the PIV method is affected by the ability of the used seeding particles and also by the way of determining the particle image shift. The aim of the work is to design and verify the new algorithm, the use of which can help to increase the measuring accuracy of the PIV method. Following the analysis and comparison of the used methods and applications of other tools of image processing, a new algorithm for evaluation of displacement based on the Image Deformation Method was designed and verified.

DESIGN OF NEW ALGORITHMS

Currently, a large amount of algorithms[1-5] can be used for data processing by the PIV method. One of the most common algorithms is the iterative calculation using the integer displacement of interrogation areas IA [5, 7-9]. Since the issue has received considerable attention in last twenty years, there are a number of computational algorithms[2-4,7-10]. Our work focuses on modification of a method known as Image Deformation Method (IMD) **Figure 1**, which is one of the most precise methods [1, 2, 7]. As the name suggests, the aim of the method is to find such a deformation function, which minimizes the resultant

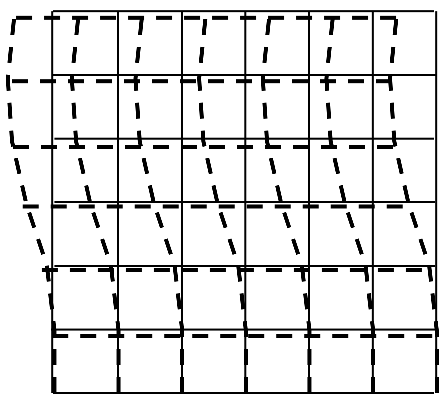

Figure 1: Example of using IDM method. Full line indicates undeformed mesh. Dashed line indicates the result of "deforming data on the basis of measured displacement"

Mutual shift of the particle images. The searched field of images shift can be then determined based on such deformation function. Based on the performed research, we decided to divide the design of the new algorithm into two separate points:

1. Improvement of the algorithm for calculation of correlation plane.
2. Design of a more precise method of calculation of the position of the center of the signal peak.

On the basis of works presented in literature[2, 3, 5, 7, 11.] We decided to combine classical iterative method with an IMD method using the B-spline order of 6. In order to determine the position of the peak center, we decided to use the following methods and to verify their accuracy:

- The use of the regression method for the calculation of the theoretical shape of the signal peak assuming that the shape of the signal peak can be replaced by symmetrical Gaussian function.
- The use of the regression method for the computation of theoretical shape of the signal peak assuming that the shape of the signal peak can be replaced by elliptical Gaussian function.

However, the results presented in professional papers on the subject[1, 2, 7] show that the method based on mutual "deformation" of individual IA is one of the most accurate methods used for processing of data measured by the PIV method. The classical algorithm using IDM is based on iterative approach and mutual deformation of individual double images. The difference to the "adaptive" correlation is that in each intermediate step the individual IA are mutually deformed based on the calculated displacement in the previous iteration. Our designed algorithm combining classical iterative procedure with the IDM method can be summarized into following points:

1. Selection of size of IA and calculation of displacement by classical method.
2. Based on calculated displacement in the previous iteration, a mutual displacement of IA is carried out and "residual" displacement is calculated. (Before the new calculation of the displacement the size of edge of IA can be reduced.)
3. Repetition of point 2 until the residual displacement is less or equal to 0.5 pixel, or until the number of repetitions does not exceed the limit determined by the user.
4. Calculation of spatial gradient of displacement in whole measured area.
5. Mutual deformation of particular IA based on calculated spatial gradient and resultant calculation of correlation plane.
6. Final detection of peak and calculation of position of peak center.

Procedure of designing an algorithm, as shown in points 1-6, needs adding a few more details. The following Taylor's development was used for the calculation of the velocity gradient:

$$\Delta x = \Delta x(x, y) = \Delta x(x_0, y_0) + \frac{\partial \Delta x}{\partial x}(x - x_0) + \frac{\partial \Delta x}{\partial y}(y - y_0) \quad [1]$$

$$\Delta y = \Delta y(x, y) = \Delta y(x_0, y_0) + \frac{\partial \Delta y}{\partial x}(x - x_0) + \frac{\partial \Delta y}{\partial y}(y - y_0) \quad [2]$$

For calculation of the mutual „deformation“ of IA, a cubic B -spline (order of six) [1] was used. In order to compare the accuracy of the designed algorithm with the classical method, the classical 3PSPI (three-point sub-pixel interpolation) was first applied for the calculation of the signal peak center. Two tests – UFT and CFT were used when testing the correct functioning of the designed algorithm. Each test was carried out for two densities of particles Ni=5 and Ni=20 for 32x32 pixels. In the UFT test, the dependence of total error on the size of the particle images within 1.9 - 5 pixels were observed, the displacement of particles was constant throughout the whole investigated area. Synthetic data were generated based on the recommendation given by Okamoto and Nishio in their work[12]. The result of UFT is shown in **Figure 2**. Compared with classical iterative procedure without „deformation“ (in case of UFT it is more appropriate to speak more about sub-pixel displacement), a significant decrease of the total error throughout the whole area of testing parameters can be observed. Particularly important is the significant insensitivity of the method to the size of images of seeding particles.

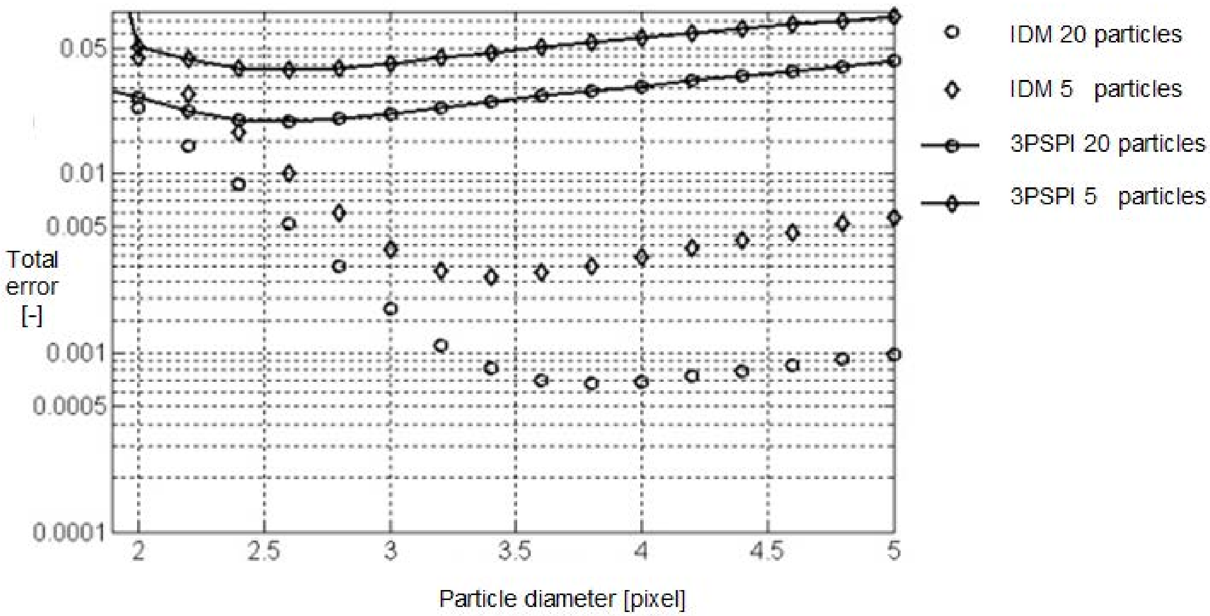

Figure 2: Results of UFT. Comparison of classical iterative procedure (3PSPI) and IDM method with the use of B-spline order of six for two densities of particles and IA=32x32 pixels

The diagram in **Figure 2** clearly shows that for particles bigger than 3 pixels, the accuracy of the method is sensitive to the size of the particle images. The accuracy of the method, however, continues to be influenced by the particle density inside IA, but not significantly. For NI=5 the accuracy for the particle diameter of D>3 pixel is around 0.005 pixels, this value is 10 times better compared with the classical method. The accuracy of the presented algorithm can be further increased when using the linear filters. The use of classical linear filter for "smoothing" of the correlation plane has proven to be the most suitable one. The filter coefficients are described by the following equation:

$$F(i,j) = \exp\left[-0.5(i^2 + j^2)\right] \quad i,j = -1;0,1. \qquad [3]$$

The diagrams in Figure 2 and Figure 3 clearly demonstrate a significant increase in accuracy when using the modified IDM method compared to classical iterative procedure. The accuracy of IDM was further increased by using the linear filter. As a result of the linear filtration of the correlation plane, the accuracy is increased by about 50% compared to the original algorithm without using the linear filtration of correlation plane **Figure 3**. It was further necessary to verify the sensitivity of the proposed algorithm to the size of the velocity gradient. For this purpose, the CFT was used. The results of CFT are shown in the following diagrams. Figure 4 indicates that tested algorithm is not sensitive to the size of the velocity gradient. CFT test was implemented for sizes of particles of 2.2 pixels. This size was chosen with regard to the results of similar tests presented by Raffel *et al.* [7]. The UFT results for the proposed IDM method clearly show that the particle size of 2.2 pixels is not optimized for this algorithm. The best accuracy can be achieved in the range of particle size of 3-5 pixels. For particles of this size, the achieved accuracy is one order better than for particles of 2.2 pixels.

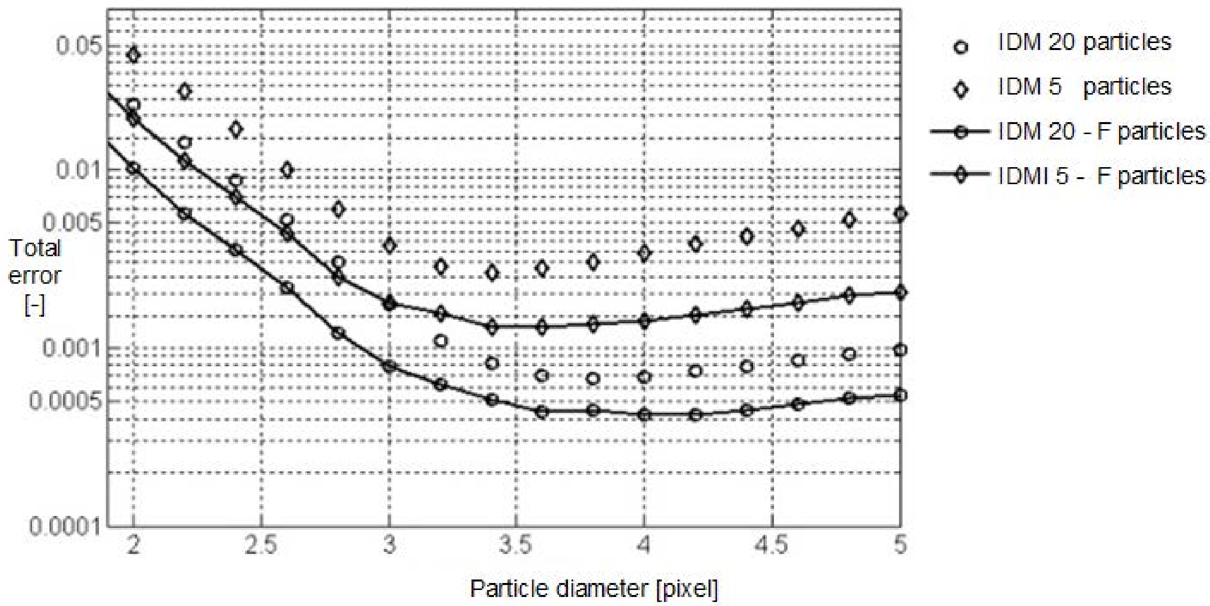

Figure 3: Results of UFT. Comparison of IDM without linear filtration and with linear filtration of correlation plane. Parameters of the test are the B-spline of order of six, two densities of particles and IA=32x32, data without noise.

The results of CFT shown in the diagram in **Figure 4** are nevertheless highly satisfactory. The size of the total error for Ni=5 is also below 0.02 pixel in the whole range of the velocity gradient. For small sizes of the velocity gradient the improvement is not so significant. The accuracy in the whole range of the particle sizes can improve if the correlation area - CA - is filtered by the linear filter described by equation 3. The comparison of the modified IDM using the filter or without it is shown in the diagram in **Figure 5.** The diagram indicates an improvement in accuracy by about 50% compared to the original algorithm without filtration of the correlation plane. The assumption was once again confirmed that the use of linear filtration of the correlation plane is a suitable tool when increasing the accuracy of the proposed algorithm. The proposed algorithm for the calculation of displacement indicates an increased accuracy compared with the classical adaptive correlation. The significant insensitivity to the size of the velocity gradient is also the advantage of the proposed procedure. The first part of the algorithm design includes designing of the modified IDM algorithm for calculation of CA. Another part includes designing of more suitable method for more accurate determination of the signal peak center. The following chapter deals with the progress of the work when designing such method.

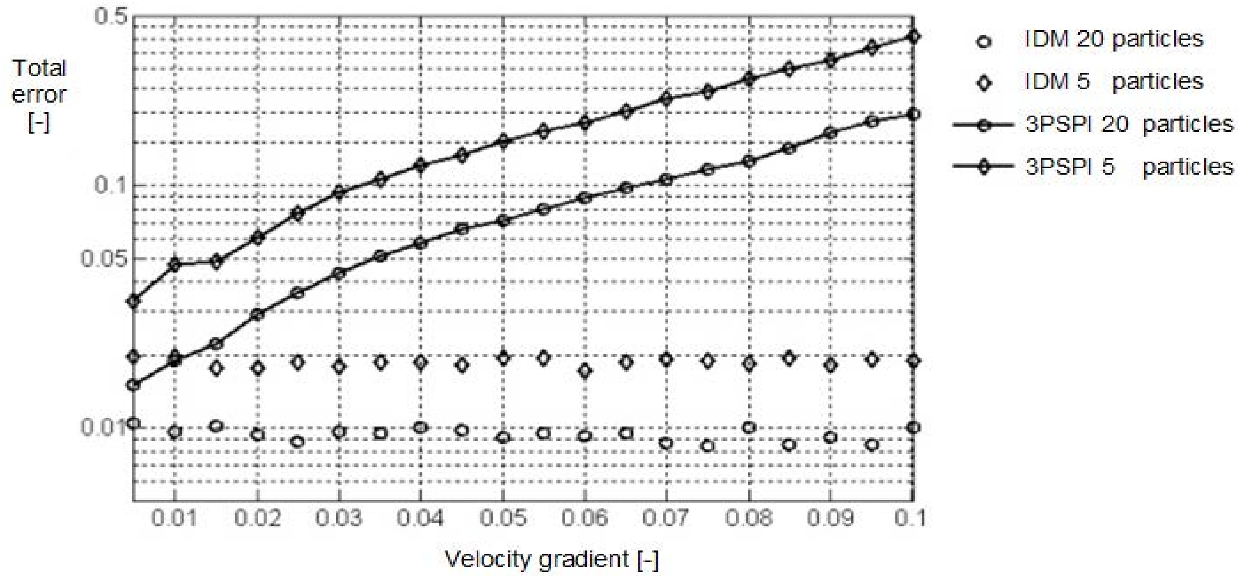

Figure 4: Results of CFT. Comparison of classical iterative procedure (3PSPI) and IDM using the B-spline order of six. Parameters of test are IA=32, number of particles Ni=5 and 10, data without added noise.

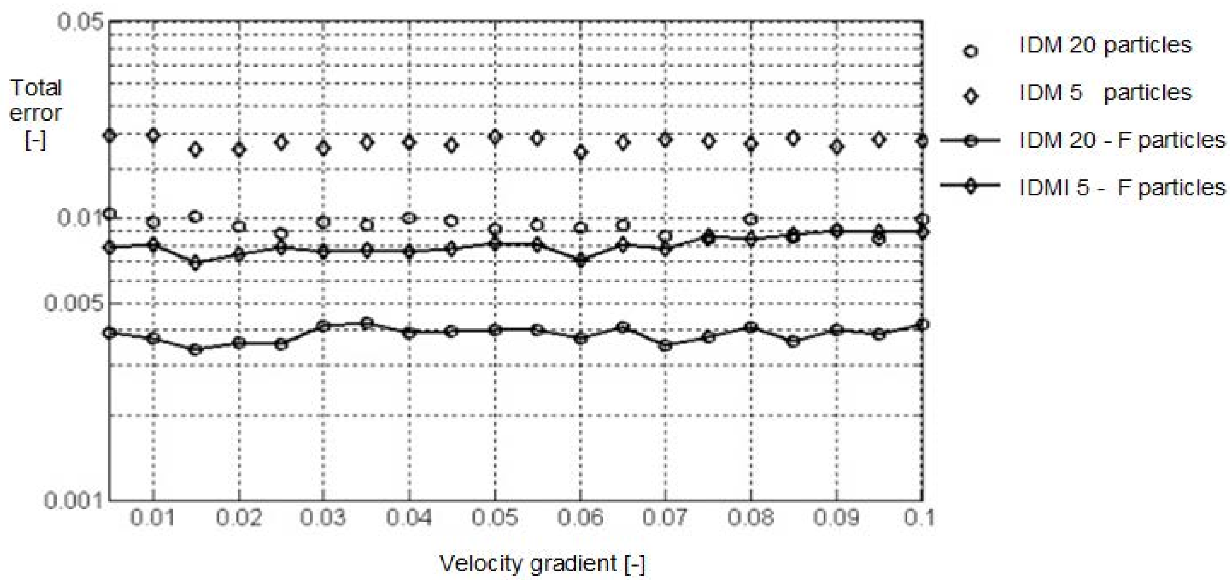

Figure 5: Results of CFT. Comparison of IDM without linear filtration and with linear filtration of correlation plane. Parameters of the test are IA=32, number of particles Ni=5 and 10, data without added noise, B-spline of order of six.

CHOICE OF THE METHOD OF DETERMINATION OF THE SIGNAL PEAK CENTER

In the large majority of commercial software the 3PSPI for determining the exact position of the signal peak is used[7]. In order to determine the signal peak center the method of regression can be used as described in Nobach's works[13]. The results presented by Nobach clearly show that using the 2D regression can help to increase the accuracy of evaluation of the measured displacement. When using regression, the shape of the signal peak can be considered not only as an elliptical Gaussian function, but also as a symmetrical Gaussian function described by equations:

$$z_{II}(i,j) = a \cdot \exp\left[b_1(i-x_0)^2 + b_2(j+y_0)^2\right]. \qquad [4]$$

Maximum of the function lies at [x0,y0]. Equation (2) can also be rewritten as:

$$z_{II}(x,y) = \exp\left[c_0 + c_1 x + c_2 x^2 + c_3 y + c_4 y^2\right], \qquad [5]$$

Where:

$$c_0 = \ln(a) + b_1 x_0^{\ 2} + b_2 y_0^{\ 2}, \qquad [6]$$

$$c_1 = -2b_1 x_0, \qquad [7]$$

$$c_2 = b_1, \qquad [8]$$

$$c_3 = -2b_2 y_0, \qquad [9]$$

$$c_4 = b_2. \qquad [10]$$

Our goal is to find, with the help of the regression, a minimum of the norm:

$$L_1 = \sum_{i,j=-1}^{1}\left[\left\{c_0 + c_1 i + c_2 i^2 + c_3 j + c_4 j^2\right\} - \ln(R_{x+i,y+j})\right] \qquad [11]$$

Where:

[x, y] is integer coordinates of maximum of the correlation plane R.

The result of the 2D regression is the following equations for coefficients c_0-c_4.

$$c_0 = 3i - j - 3, \qquad [12]$$

$$c_1 = 6i^2, \qquad [13]$$

$$c_2 = -2i + j + 2, \qquad [14]$$

$$c_3 = 6j^2, \qquad [15]$$

$$c_4 = -2i + 3. \qquad [16]$$

From 12-16 equations the position of the peak center and height can be expressed:

$$x_0 = -\frac{c_1}{2c_2},\ y_0 = -\frac{c_3}{2c_4},\ a = \exp(c_0 - c_2 x_0^{\ 2} - c_4 y_0^{\ 2}). \qquad [17]$$

Before using the regression, the linear filter can be used with the help of which the accuracy should be increased. For this reason, the use of the following linear filter defined by equation (1) was proposed. The symmetrical Gaussian function was used for testing of the suitability of the 2D regression. Two sizes of areas - 3x3 and 5x5 points - were considered for 2D regression. For each of these areas, weighting function was used. The following weighting function was used for the area of 3x3 points and in the area of 5x5:

$$W = \begin{bmatrix} 0.722 & 0.833 & 0.722 \\ 0.833 & 1 & 0.833 \\ 0.722 & 0.833 & 0.722 \end{bmatrix}, \qquad W = \begin{bmatrix} 0.853 & 0.912 & 0.912 & 0.912 & 0.853 \\ 0.912 & 1 & 1 & 1 & 0.912 \\ 0.912 & 1 & 1 & 1 & 0.912 \\ 0.912 & 1 & 1 & 1 & 0.912 \\ 0.853 & 0.912 & 0.912 & 0.912 & 0.853 \end{bmatrix} \qquad [18]$$

In order to verify the accuracy of the proposed methods, the UFT test was applied again, in which the influence of the accuracy of the size of the seeding particle images was observed. Density of particles was another observed parameter. The test was carried out for two densities of particles, 5 and 20, in the area of 32x32 pixels. For comparison, the reference dependence for classical 3-point subpixel interpolation is always given. The influence of using the linear filter was also tested. The test results are shown in the following diagrams, Figure **6 - 9**.

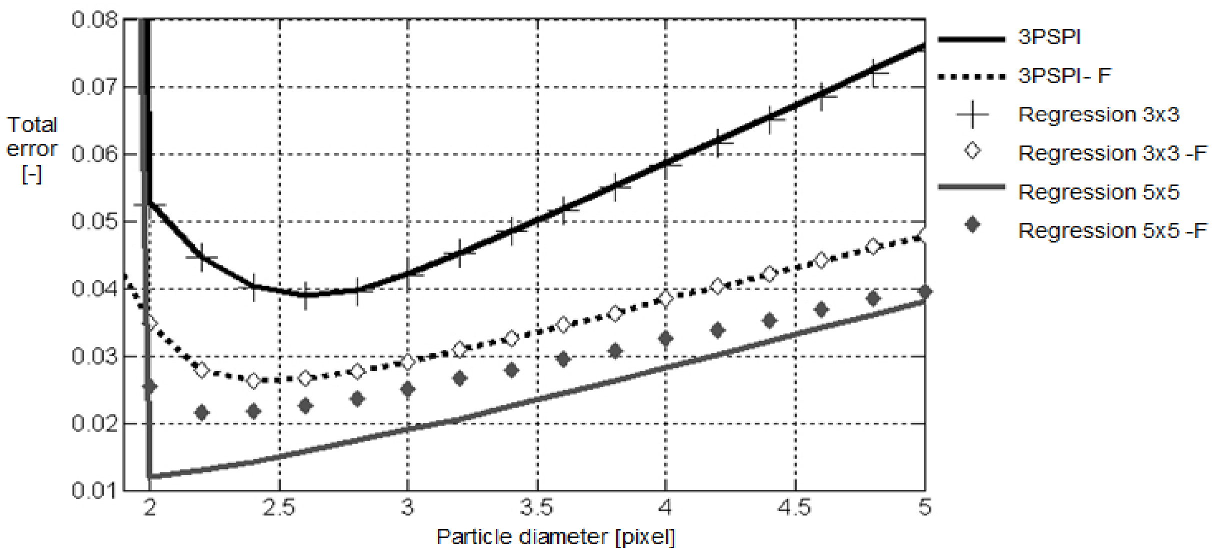

Figure 6: Test of accuracy of 2D regression, assuming the elliptical shape of signal peak. Comparison with classical 3-point subpixel interpolation. A test was carried out for Ni=5, area size of 32x32. The marking F at the end means that before the performance of the calculation of the displacement a linear filter was applied to the correlation plane.

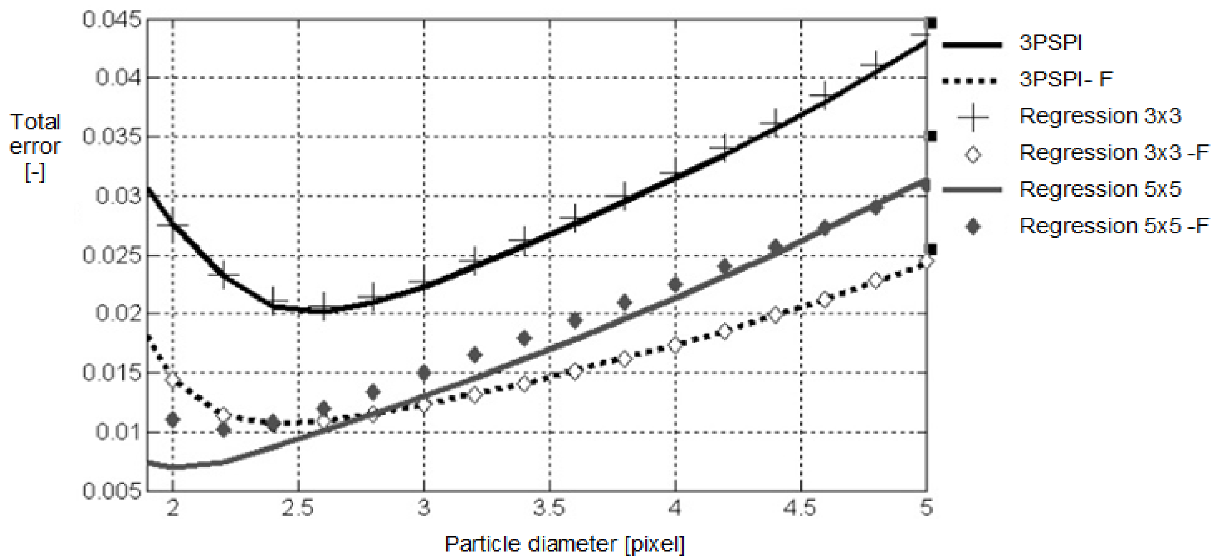

Figure 7: Test of accuracy of 2D regression, assuming the elliptical shape of signal peak. Comparison with classical 3-point subpixel interpolation. A test was carried out for Ni=20, area size of 32x32. The marking F at the end means that before the performance of the calculation of the displacement a linear filter was applied to the correlation plane.

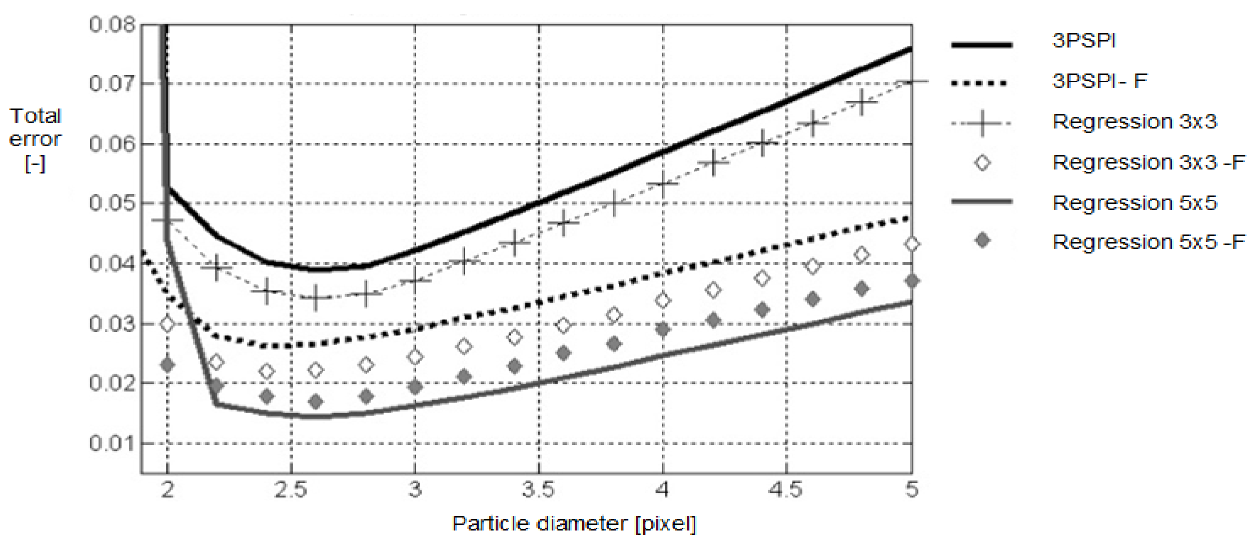

Figure 8: Test of accuracy of regression assuming symmetrical shape of signal peak. Comparison with classical 3-point subpixel interpolation. A test was carried out for Ni=5, area size of 32x32. The marking F at the end means that before the performance of the calculation of the displacement a linear filter was applied to the correlation plane.

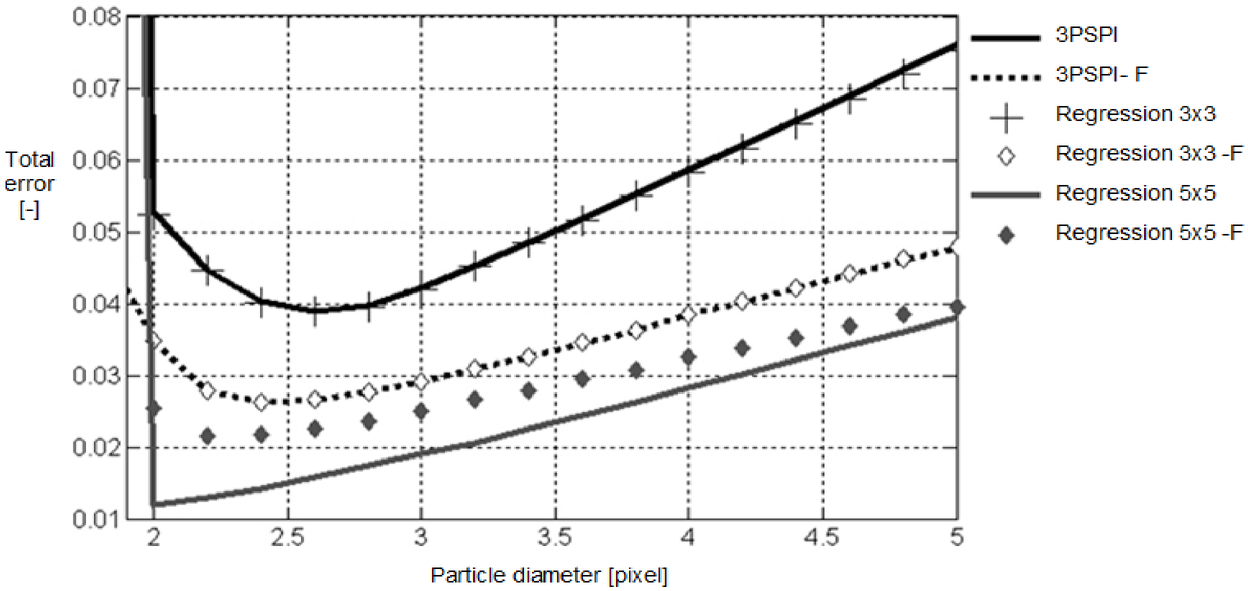

Figure 9: Test of accuracy of regression assuming symmetrical shape of signal peak. Comparison with classical 3-point subpixel interpolation. A test was carried out for Ni=20, area size of 32x32. The marking F at the end means that before the performance of the calculation of the displacement a linear filter was applied to the correlation plane.

The comparison of particular diagrams shows that the use of the regression method together with replacement of the peak by the elliptical Gaussian function is not suitable. With the density of particles Ni=5 the method is more accurate, but on the other hand, with density Ni=20, the method shows significantly worse accuracy, even in comparison with the classical 3-point subpixel interpolation. On the contrary, the linear filter can be used with an area size of 3x3, whereby the accuracy is increased by 25% - 50% depending on the number of particles inside IA.

On the other hand, the increase in accuracy when using regression and symmetrical Gaussian function is about 50% compared with 3PSPI without using the linear filter. The increase in accuracy compared with 3PSPI with filter is about 20%. The results presented show that the use of the regression method with an area size of 3x3 pixels and suitable linear filter helps to increase the accuracy of the evaluated displacement. The UFT was performed for a zero noise level. The real data, however, always contain a certain level of noise depending on the quality of the used experimental equipment and the experiment. The noise level together with other parameters significantly influence the measuring accuracy. For this reason, it was necessary to find out how the chosen algorithm is sensitive to the noise level inside IA. The test results are shown in **Figure. 10 and 11.** These diagrams clearly show that the 2D regression method, as designed, is very insensitive to the noise level of less than 5%. With higher level of noise this method indicates an increased inaccuracy for particles sizes smaller than 3 pixels and for low values of Ni. Proceeding from the definition of noise, as presented by [14], there is a possibility how to remove, at least partially, the aforementioned noise from the measured data. If we identify the maximum of a histogram of the grayscale in the relevant IA, then such value can be deducted from the original IA and values lower than 0 put equal to zero. If we remove the noise from IA in this manner and perform the sensitivity test again, we obtain changed dependencies compared to those in **Figure 12 –and 13.**

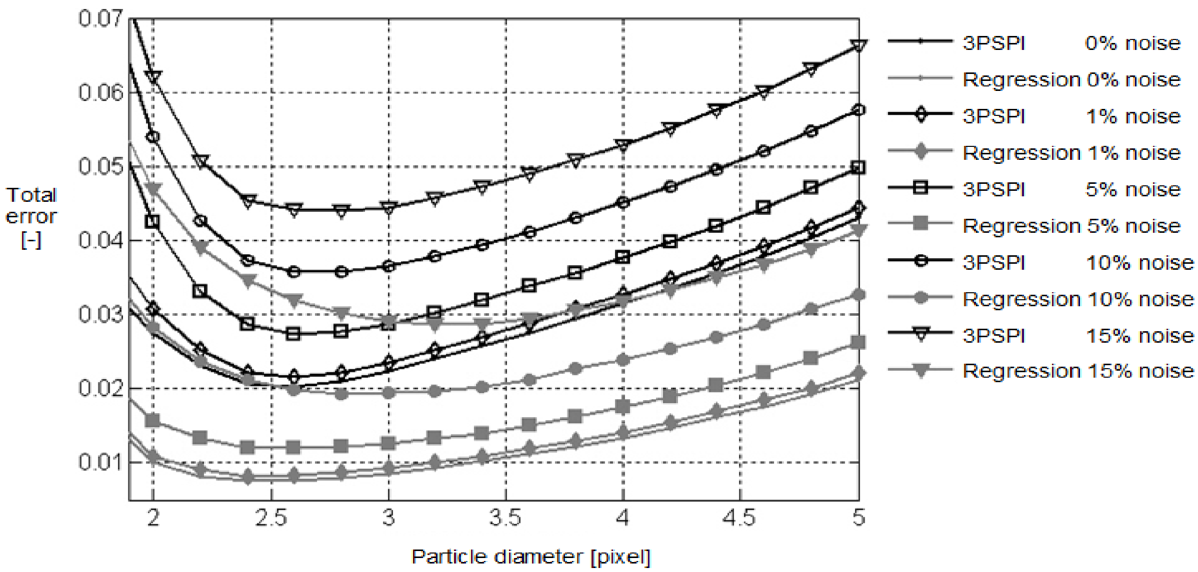

Figure 10: Influence of sensitivity to noise level. Symmetrical shape of signal peak and 9-point regression. A test was carried out for Ni=20, area size of 32x32. The correlation plane was not filtered by linear filter. Comparison with classical 3PSPI.

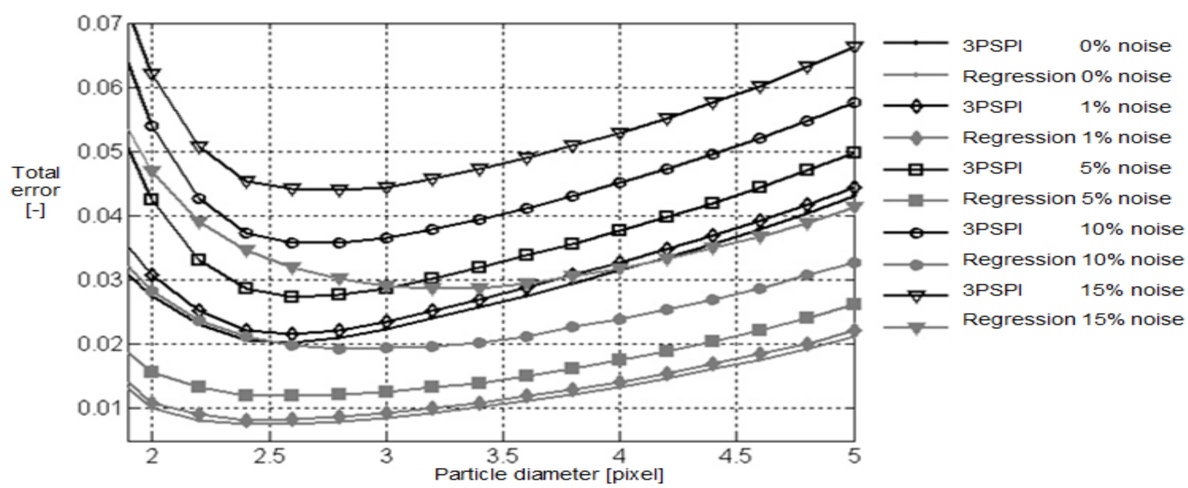

Figure 11: Influence of sensitivity to noise level. Symmetrical shape of signal peak and 9-point regression. A test was carried out for Ni=20, area size of 32x32. The correlation plane was not filtered by linear filter. Comparison with classical 3PSPI.

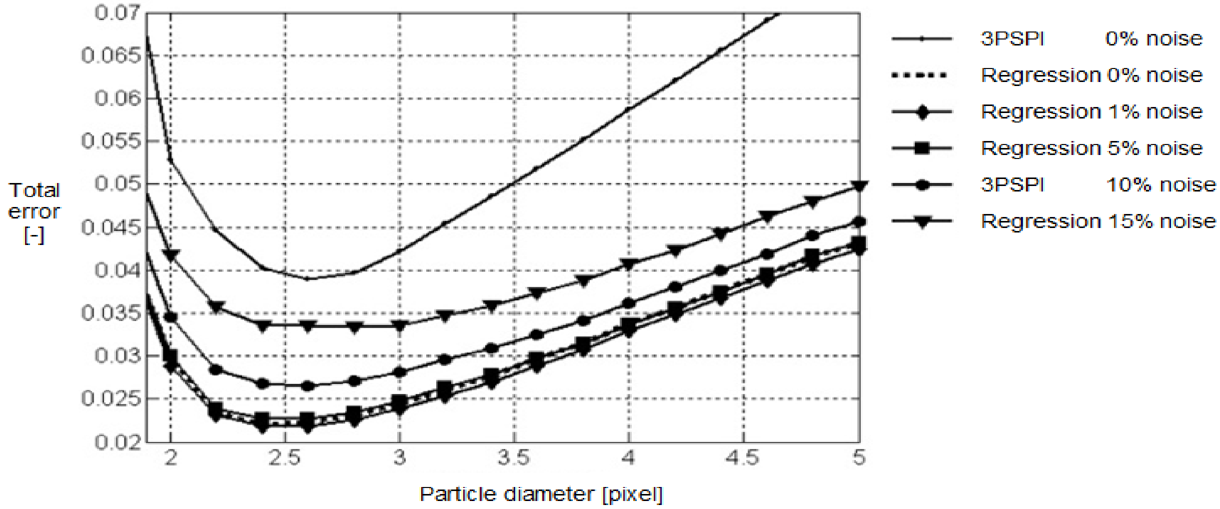

Figure 12: Influence of sensitivity to noise level. The noise before performing the correlation plane calculation was detected and partially removed with the help of the threshold value. Symmetrical shape of signal peak and 9-point regression. A test was carried out for Ni=5, area size of 32x32. The correlation plane was filtered by linear filter. Comparison with classical 3PSPI.

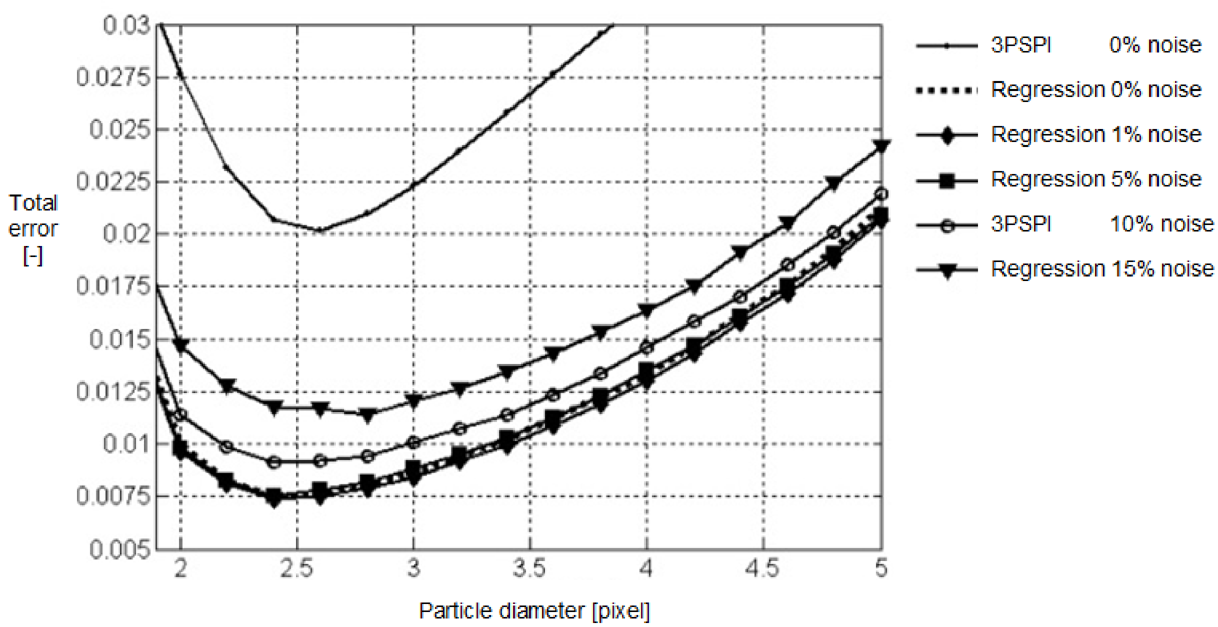

Figure 13: Influence of sensitivity to noise level. The noise before performing the correlation plane calculation was detected and partially removed with the help of the threshold value. Symmetrical shape of signal peak and 9-point regression. A test was carried out for Ni=20, area size of 32x32. The correlation plane was filtered by linear filter. Comparison with classical 3PSPI.

The results of the tests of sensitivity to noise presence inside the IA after its elimination indicate a significant increase in the accuracy of the measured data. The resultant measuring accuracy indicates, after noise detection and its elimination, only slight sensitivity to noise level of less than 10%. The increase in accuracy occurs also in case of low number of particles and high noise level. The presented results clearly show that when using the regression method the detection and elimination of the noise level of the measured data must be used. If the experimenter uses the regression method without this procedure, it may happen that in some cases the measured displacement will indicate a higher degree of uncertainty than when using the classical method of 3PSPI.

CONCLUSION

On the basis of the previous analysis, a new algorithm based on the IDM method and using a 2D regression for the calculation of the signal peak center position was designed, programmed and tested **Figure 14 and 15**. Based on the performed analysis the final algorithm can be summarized into the following steps

- To use classical iterative procedure using the integer displacement in individual intermediate steps
- To use modified IDM algorithm in the last step of the iteration
- To detect the noise level inside the IA and subsequently eliminate it with the help of the threshold value.
- To use the linear filter for "smoothing" of the correlation plane before the calculation of the displacement is performed.
- To use a 9-point 2D regression for the calculation of the center position of the signal peak in the correlation plane on the condition of the peak shape approximation of axially symmetrical Gaussian functions.

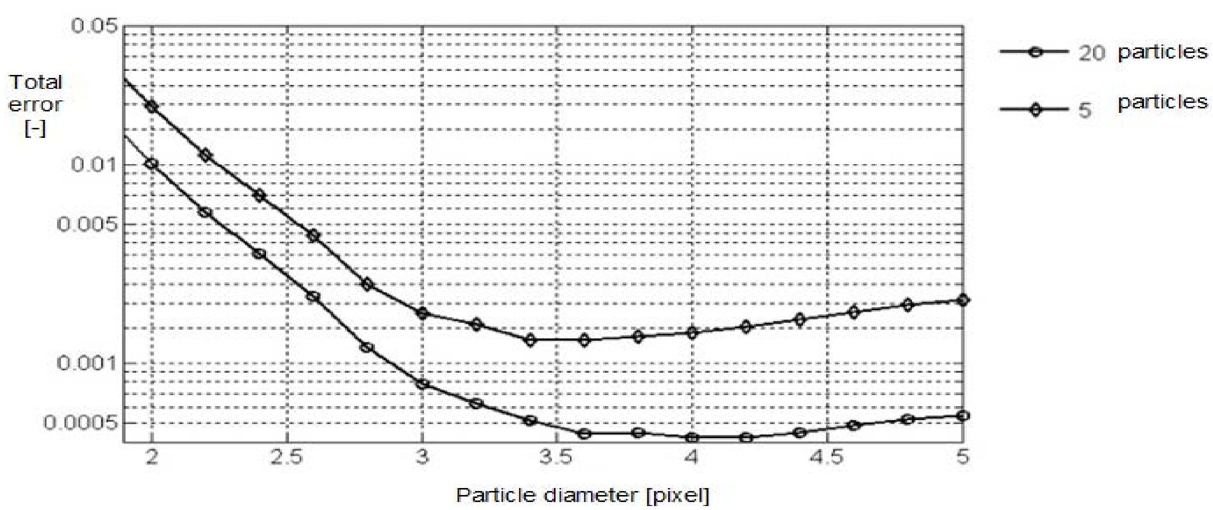

Figure 14: Result of UFT for modified IDM method, test of sensitivity to the particle diameter size, constant displacement of 0.5 pixels, IA=32x32 pixels, data without noise. The test was carried out for two densities of particles Ni=5 and Ni=20.

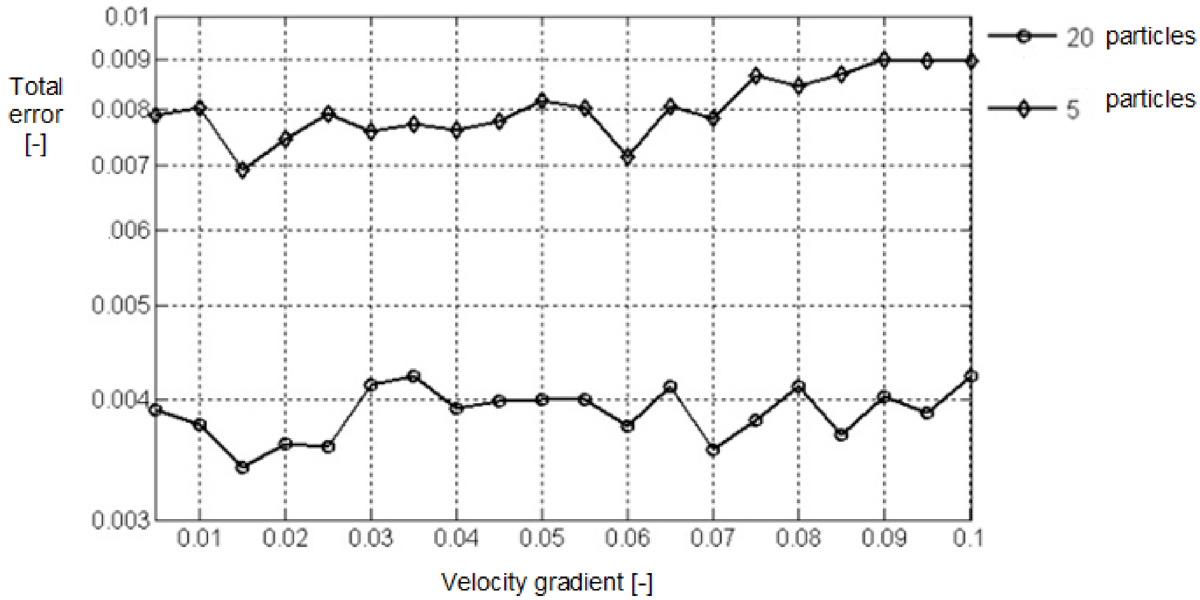

Figure 15: Result of CFT for modified IDM method, test of sensitivity to the velocity gradient size, IA=32x32 pixels, data without noise. The test was carried out for two densities of particles Ni=5 and Ni=20.

The use of the algorithm composed of the above steps significantly increases the measuring accuracy of the PIV method. The decrease of the total error of the measurement of such proposed algorithm manifests itself more strongly for particles with a diameter of 2.5 pixels and more. In this case, the total error falls to a tenth of its original value. For smaller particles, the decrease of the total error is significantly smaller. The proposed method significantly reduces the total measurement error, especially in the event of large velocity gradient inside IA. The proposed algorithm demonstrates the constant value of the total error within the whole range of testing values of the gradient. The total error is in such a case about 5 times smaller than in the case of classical iterative algorithm and depends only on the particles density and size. The next advantage is the significant insensitivity of such algorithm to the size of the velocity gradient inside the IA. Another advantage is that the detection and subsequent elimination of the noise helps to significantly suppress the negative impact of noise on the measurement accuracy. This method is so efficient that the negative impact of the noise is suppressed for the noise level up to 5%. At higher values, the measurement error when using this filter is lower by about 70%.

The proposed algorithm was tested using the synthetic data and the range of tests parameters was selected in the area that is normally considered to be optimal for PIV. Beyond the limits tested, the algorithm may show worse results compared with classical iterative procedure. Based on previous results, it should be noted that the proposed algorithm is not suitable for analysis of data containing particles with smaller diameters of 2.2 pixels and less. It should also be observed that the whole work deals only with the processing of recorded data and minimization of the measurement error caused by their evaluation. This or any other evaluation procedure cannot affect in any way the other measurement errors caused by incorrect setting of the measurement system or by unsuitable optical conditions during the experiment, and by other errors that could adversely affect the resultant measuring accuracy. The presented results clearly show that the proposed procedure of evaluation significantly increases the measuring accuracy, even under conditions where the accuracy of the common algorithm has been insufficient due to the present noise. Since the algorithm has been tested only on synthetic data, its function needs to be verified in the real data as well. This part of the work remains to be done and we hope the results will confirm our current findings.

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