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Research Notes

Time Series Analysis of Kaprekar's (Constant) Number

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ABSTRACT

It is observed that Kaprekar discovered a constant which was named after him. The number of steps required to arrive at the Kaprekar's number for all the four digit numbers change randomly. The Rescale Range Analysis of Sequence of these random numbers is carried out and interesting results are presented.

Keywords: Fractal, Time Series, R/S Analysis, Hurst Exponent.

INTRODUCTION

Shri Dattathreya Ramachandra Kaprekar was born on January 17, 1905 in Dahanu which is near Mumbai, India. Recreational math became his hobby as a child he enjoyed spending time solving math puzzles and problems. In 1946 he discovered Kaprekar's Constant which was named after him. The Constant is 6174. This number is arrived after few iterations of a given procedure for all the four digit numbers. It is found that the number of steps required arriving at the kaprakar's number, starting with a given four digit number exhibits random character. This set of random number is analysed using rescale range analysis technique and fractal dimension[1, 2] is also calculated.

What is the Kaprekar Number?

The number 6174 is called the Kaprekar number[3, 4]. The Indian mathematician D.R.Kaprekar made the following discovery in 1949.

(1) Take a four-digit number with different digits (acbd with .$a<b<c<d$)..

(2) Form the largest and the smallest number from these four digits (dcba and abcd)..

(3) Find the difference of these digits. Maybe this is 6174 (dcba - abcd = 6174?).

If it is not, form the largest and the smallest number from the difference and subtract these numbers again. You may have to repeat this procedure.

The end result is always 6174, but there are no more than 7 steps.

(4) 1^{st} example: Take the number 1746.

1^{st} step: 7641 - 1467 = 6174

(5) 2^{nd} example: Take the number 5644.

1^{st} step: 6544 - 4456 = 2088

2^{nd} step: 8820 - 0288 = 8532

3^{rd} step: 8532 - 2358 = 6174

(6) 3^{rd} example: Take the number 7652.

1^{st} step: 7652 - 2567 = 5085

2^{nd} step: 8550 - 0558 = 7992

3^{rd} step: 9972 - 2799 = 7173

4^{th} step: 7731 - 1377 = 6354

5^{th} step: 6543 - 3456 = 3087

6^{th} step: 8730 - 0378 = 8352

7^{th} step: 8532 - 2358 = 6174

(7) The problem is solved. (Spektrum der Wissenschaft, Erstausgabe 1978)

Manick Srinivasan and Ramkumar Ramamoorthy sent me more computer results: If you allow numbers with noughts at the beginning like 0342 or 0045, then the Kaprekar number fits for any four digit number except for 1111, 2222, 3333, 4444, 5555, 6666, 7777, 8888, 9999.

Hurst's Rescale Range Analysis: The approach to the Quantification of correlations in time series was developed by H. E. Hurst. He spent his life studying the hydrology of the Nile River, particularly the record of floods and droughts .He considered the river flow as a time series and determined the storage limits in an idealized reservoir. Hurst developed a new exponent called Hurst Exponent (H). The Hurst exponent[5, 6] ' H' can classify time series into random and non random time series. Hurst exponent is also related to the fractal dimension. There are different techniques for the estimation of Hurst Exponent. R/S is a method used for distinguishing completely random time series from a correlated time series. In this analysis Average of given Sequence of observations Z (t) is taken first

$$\langle Z \rangle_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} Z(t) \qquad \text{... 1}$$

$$X(t,\tau)=\sum_{t=1}^{\tau}\left(Z(t)-\langle Z\rangle_{\tau}\right) \qquad \text{... 2}$$

The self adjusted range R (τ) is defined as the difference between maximum and minimum accumulated influx X

$$R(\tau) = \max X(t, \tau) \ - \ \min X(t, \tau) \qquad \text{... 3}$$

Hurst used a dimension less ratio R/S where s (τ) is the standard deviation as a function of τ is

$$S(\tau)=\left(\frac{1}{\tau}\sum_{t=1}^{\tau}\left(Z(t)-\langle Z\rangle_{\tau}\right)^{2}\right)^{\frac{1}{2}} \qquad \text{... 4}$$

and found that observed rescaled range for a time series is given as

$$(R/S_{)} = (\tau / 2)^{H} \qquad \text{... 5}$$

Plotting log (R/S) against log (τ /2), the Hurst exponent H can be found as the slope of the resulting straight line.

If H is between 0.5 and 1, the trend is persistent which indicates long memory effects. This also means that the increasing trend in the past implies increasing trend in the future also or decreasing trend in the past implies decreasing trend in the future also. In contrast to this, if *H* is between 0 and 0.5 then an increasing trend in the past implies a decreasing trend in the future and decreasing trend in past implies increasing trend in the future. It is important to note that persistent stochastic processes have little noise whereas anti-persistent processes show, presence of high frequency noise. The relationship between fractal dimensions D_f and Hurst exponent H can be expressed as [16]

$$D_f = 2 \ - H \qquad \text{... 6}$$

From the Hurst exponent H of a time series, the fractal dimension of the time series can be found. When D_f = 1.5, there is normal scaling. When D_f is between 1.5 and 2, time series is anti-persistent and when D_f is between 1 and 1.5 the time series is persistent. For D_f =1, time series is a smooth curve and purely deterministic in nature and for D_f = 1.5 time series is purely random. Long term correlations of indexes in developed and emerging markets have been studied by using Hurst analysis.

DATA ANALYSIS

From equation (1) to (4) We have a value of (R_N / S_N) for the time series, X(t). t=1, 2, 3….N. Since we are interested in how (R / S) Varies with successive subintervals τ of N, we substitute τ for N in equation (1) to (4) .the Hurst exponent is obtained from equation

$$(R_{\tau} / S_{\tau})_{Ave} = (\tau / 2)^{H} \qquad \text{... 7}$$

for example, if 64 value of X(t) are available for time series , the R_N and S_N for N=64 are obtained .then data are broken into two parts , each with τ = 32 (1,2 …32 and 33,34…64). the value for R_{32} and S_{32} are obtained for the two parts . the two values of R_{32} / S_{32} are then averaged to give $(R_{32} / S_{32})_{ave}$.the data set is then broken in four parts, each with τ =16(1,2,…16;17,18…32;33,34…48;and49,50…64) .the value of (R_{16} / S_{16}) are obtained for the four parts and are averaged to give $(R_{16} / S_{16})_{ave.}$ this process is continued for $\tau = 8$ and 4 to give $(R_8 / S_8)_{ave}$ and $(R_4 / S_4)_{ave}$ for $\tau = 2$, the vlue for $R_2 = S_2$ so that $(R_2 / S_2) =1$. the values of log $(R\tau / S\tau)_{ave}$ are plotted against log $(\tau /2)$ and the best fit straight line gives H from equation (7)

Rescale Range Analysis of Kaperaker 'S Number

The following **Table: 1** shows data range, Hurst Exponent, Fracatal dimension and trends To study the scaling behavior of the number of steps required to arrive at the Kaprekar number, starting with different four digit numbers in the range of 1111 to 9999, the sequence of number of steps is divided into groups of 2048, thus forming four groups. These groups when analysed using Rescale ragne technique showed that for the first set of numbers starting from 1112 , the Hurst exponent is 0.175 and the fractal dimension is 1.825. For the remainig set of numbers, the Hurst exponent is close to 0.5 and the fractal dimension is nearly 1.5. Table 1 shows the Hurst exponent and the fractal dimension for the four different groups of number of steps required to arrive at the Kaprekar's number.

Table: 1

Set	Data Range	Hurst Exponent	Fractal dimension	Trend
1	1 to 2048	0.175	1.825	Anti-persistent
2	2049 to 4098	0.5	1.5	persistent
3	4099 to 6147	0.487	1.513	persistent
4	6148 to 8196	0.516	1.484	persistent

It is observed that the Hurst exponent is on the lower side and the fractal dimension is on the higher side for the first set of numbers. This also shows an anti-persistent behavior and relatively less variation in the number of steps. The remaining sets indicate a higher Hurst exponent and a relatively lower fractal dimension (nearly 1.5) indicating that the power law holds and the complexity associated with the variation in the number of steps is relatively less for the rest of the sets. Also the trend shown in these sets is persistent.

4. CONCLUSION

It is observed that the number of steps required arriving at the Kaprekar's number for all the four digit numbers change randomly. The sequence of these random numbers exhibit scaling character as is seen from the four plots of log(R/S) versus log (Tau/2). The initial set of numbers shows a lower value of Hurst exponent and a higher value of fractal dimension indicating anti-persistent trend. Whereas the remaining set of numbers yield a value of Hurst exponent close to 0.5 and a fractal dimension of about 1.5 indicating a persistent trend.

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