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Five Dimensional LRS Bianchi Type-I String Cosmological Model in Lyra Manifold

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Abstract: Five dimensional LRS Bianchi type-I string cosmological model is investigated in Lyra geometry. Considering the suitable form of the gauge function, which fit in observation, a cosmological model is obtained. Some physical and kinematical implications of model are also discussed.

Keywords: LRS Bianchi type-I, Lyra Manifold, String Cosmology.

INTRODUCTION

The string theory was developed to describe the events of the universe at the early stages of its evaluation therefore the study of string theory is important in the evaluation of the universe. Lyra[1], Brans-Dicke[2], Weyl[3] are some important scalar tensor alternative theories of gravitation who proposed modification of Riemannian manifold. Lyra introduced a gauge function into the structure less geometry. Further; Sen[4], Sen and Dunn[5], Halford[6, 7] Beesham[8, 9], Bhamra[10], Soleng[11] have investigated various cosmological models in a framework of Lyra's manifold. Jeavons *et al.*[12] pointed out that the Einstein field equations proposed by Sen and Dunn are very much useful based on Lyra's manifold. Zel'dovich[13], Kibble[14], Everett[15], Vilenkin[16] investigated string cosmological models and described that cosmic strings act as gravitational lenses and may give rise to density perturbations leading to formation of galaxies. Krori *et al.*[17] studied Bianchi type cosmologies in presence of cosmic strings and described the matter and strings coexist throughout the evaluation of the universe. Recently various authors Singh and Singh[18, 19], Singh *et al.*[20], Rahaman *et al.*[21-25], Mohanty *et al.*[26, 27], Rahaman and Bera[28], Kalyanshetti and Waghmode[29], Reddy and Rao[30, 31], Bali and Chandnani[32, 33], Chakraborty and Ghosh[34] , Benerjee *et al.*[35], Pradhan *et al.*[36-38], Bali *et al.*[39-42], Tikeker and Patel[43], Wang[44], Ram and Zeyauddin[45], have discussed various aspects of cosmological models in Lyra's geometry. Venkateswarlu and Pavan Kumar[46], Mohanty and Samanta[47], Rathore and Mandawat[48] have investigated higher dimensional string cosmological models in various alternative theories of gravitation and obtained exact solution of the field equations. Samanta *et al.*[49] have investigated five dimensional Bianchi type-I cosmological models generated by a cloud of string with particles attached to them in Lyra's manifold. Letelier[50] discussed massive strings and obtain exact solution for Bianchi type I and Kantonwski-Sachs space time.

In this chapter we have investigated five dimensional LRS Bianchi type-I string cosmological model in Lyra geometry. To get deterministic solution we have consider the gauge function $\beta = \frac{2}{3t}$ which is the best suitable form for fit in the observation. Some physical and kinematical parameters of the models are also discussed.

THE METRIC AND FIELD EQUATIONS

The Five dimensional LRS Bianchi type I metric is consider as

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) + C^2d\xi^2 \tag{1}$$

Where $A, B,$ and C are functions of cosmic time $'t'$ only.

The field equations based on Lyra Manifold in normal gauge proposed by Sen and Sen and Dunn is written as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -\chi T_{ij} \tag{2}$$

Where, $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is Einstein tensor, T_{ij} is the stress energy of matter, ϕ_i is the displacement vector which is given by

$$\phi_i = (\beta(t), 0,0,0,0) \tag{3}$$

The energy momentum tensor T_i^j for cosmic string is taken as

$$T_i^j = \rho u_i u^j - \lambda x^i x_j \tag{4}$$

Where, ρ and λ are the rest energy density and the tension density of cloud of strings respectively, u^j is the cloud velocity and x^i is the direction of string i.e. direction of anisotropy satisfies the relation

$$u^i u_i = -x^i x_i = -1 \ \text{ and } \ u^i x_i = 0 \tag{5}$$

Let consider, $\rho = \rho_p + \lambda$ be the energy density of cloud of strings with particle attached to them, ρ_p be the rest energy density of particles.

Here ρ and λ are functions of cosmic time t only.

For the line element (1) the components of energy momentum tensor T_i^j are given as

$$T_0^0 = -\rho, \quad T_1^1 = -\lambda, \quad T_2^2 = 0, \ T_3^3 = 0, \quad T_4^4 = 0$$

$$\text{And} \quad T_i^j = 0 \quad \text{when } i \neq j \tag{6}$$

The trace of energy momentum tensor T is given by

$$T = T_0^0 + T_1^1 + T_2^2 + T_3^3 + T_4^4$$

$$T = -(\rho + \lambda) \tag{7}$$

The Lyra Manifold's field equations for the line element (1) by using equations (3) to (7) in equation (2) is obtained as

$$\frac{2A_4B_4}{AB} + \frac{2B_4C_4}{BC} + \frac{A_4C_4}{AC} + \frac{B_4^2}{B} - \frac{3}{4}\beta^2 = \chi\rho \tag{8}$$

$$\frac{2B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{B_4^2}{B} + \frac{3}{4}\beta^2 = \chi\,\lambda \tag{9}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} + \frac{3}{4}\beta^2 = 0 \tag{10}$$

$$\frac{A_{44}}{A} + \frac{2B_{44}}{B} + \frac{2A_4B_4}{AB} + \frac{B_4^2}{B} + \frac{3}{4}\beta^2 = 0 \tag{11}$$

Here suffix '4' denote the ordinary differentiation with respect to cosmic time 't'.

SOLUTION OF THE FIELD EQUATIONS

In the order to derive exact solutions of the field equations (8) to (11) we consider the following assumption

$$A = B = t^n \text{ and } C = t^m \tag{12}$$

Where n and m are arbitrary constants.

Here we have four independent field equations in six unknowns.

And since for gauge function we have no independent field equation, therefore here we consider

$$\beta = \frac{2}{3t} \tag{13}$$

which is the best suitable form for fit in the observation.

Now using equations (12) and (13) in the field equations (8) to (11), the field equations are reduces to

$$3n^2 + 3nm - \frac{1}{3} = \chi\rho t^2 \tag{14}$$

$$3n^2 + m^2 + 2nm - 2n - m + \frac{1}{3} = \chi\,\lambda t^2 \tag{15}$$

$$3n^2 + m^2 + 2nm - 2n - m + \frac{1}{3} = 0 \tag{16}$$

$$6n^2 - 3n + \frac{1}{3} = 0 \tag{17}$$

Solving equations (17) we obtain

$$n = \frac{1}{3} \text{ or } n = \frac{1}{6} \tag{18}$$

Using $n = \frac{1}{3}$ in equation (16) we get

$$m = 0 \text{ or } m = \frac{1}{3} \tag{19}$$

And

Using $n = \frac{1}{6}$ in equation (16) we get

$$m = \frac{1}{2} \text{ or } m = \frac{1}{6} \tag{20}$$

Equation (12) shows that the extra dimension contract to a Planckian length as t tends to ∞ if $m < 0$ and the universe expand indefinitely as t increase if $n > 0$.

Hence to obtain a physically realistic string cosmological model, we consider

$$n = \frac{1}{3} \text{ and } m = 0 \tag{21}$$

Using equations (12) and (21) in equation (1),

The metric (1) leads to the form

$$ds^2 = -dt^2 + t^{\frac{2}{3}}(dx^2 + dy^2 + dz^2) + d\xi^2 \tag{22}$$

SOME PHYSICAL AND KINEMATICAL PARAMETERS

For model (22), Using equations (21) and (14), we obtain the rest energy density ρ as

$$\rho = 0 \tag{23}$$

Using equations (21) and (15), we obtain the string tension density λ as

$$\lambda = 0 \tag{24}$$

The rest energy density of cloud of strings with particle attached to them $\rho_p = \rho - \lambda$ is given by

$$\rho_p = 0 \tag{25}$$

The scalar expansion (θ), spatial volume (V) and Deceleration parameter (q) for the model (22) are given by

$$\theta = u^{\mu}_{;\mu}$$

$$\theta = \frac{1}{t} \tag{26}$$

The spatial volume is given by

$$V = \sqrt{-g}$$

$$V = t \tag{27}$$

And the deceleration parameter is given by

$$q = -\frac{3}{\theta^2}\left[\theta^{i}_{;i} + \frac{1}{3}\theta^2\right]$$

$$q = 2 \tag{28}$$

The non vanishing components of shear tensor (σ_i^j) and shear (σ) for the model (22) are given as

$$\sigma_1^1 = 0 \tag{29}$$

$$\sigma_2^2 = 0 \tag{30}$$

$$\sigma_3^3 = 0 \tag{31}$$

$$\sigma_4^4 = -\frac{1}{3t} \tag{32}$$

And $\sigma^2 = \frac{1}{2}\sigma_{\mu\upsilon}\sigma^{\mu\upsilon}$

$$\sigma^2 = \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2]$$

$$\sigma^2 = \frac{1}{18t^2} \tag{33}$$

And also we have

$$\mathrm{Lim}\frac{\sigma^2}{\theta^2} \neq 0 \tag{34}$$

CONCLUSION

In this Chapter we have investigated five dimensional LRS Bianchi type-I string cosmological model in the framework Lyra geometry. It is observed that the energy density ρ and string tension density λ becomes zero therefore we obtained higher dimension vacuum model in Einstein's theory. The spatial volume V becomes zero when t tends zero and volume becomes infinite when t tends to infinite for n > 0. The scalar of expansion θ tends to zero when t tends to infinite and it tends to infinite as t tends to zero. It indicates that the model starts with a big-bang and stops at $t = \infty$. Since $q > 0$, therefore model is not inflationary. Shear σ decreases as time increases. Since $\frac{\sigma}{\theta} \neq 0$, therefore model is anisotropic.

REFERENCES

1. G. Lyra, Math. Z., 1951, 54, 52.
2. H. Brans, R.H. Dicke, Physical Review A, 1961, 124, 925-935
3. H. Weyl, Sber preuss. Acad. Wiss. Berlin, 1918, 465.
4. D.K. Sen, Z. Phys., 1957, 149, 311.
5. D.K. Sen and K.A. Dunn, J. Math. Phys., 1971, 12, 578.
6. W.D. Halford, J. Phys., 1970, 23, 863.
7. W.D. Halford, J. Math. Phys., 1972, 13, 1699.
8. A. Bheesham, Astrophys. Space Sci., 1986, 127, 189
9. A. Bheesham, Aust. J. Phys., 1988, 41, 833.
10. K.S. Bhamra, Aust. J. Phys., 1974, 27, 541.
11. H.H. Soleng, Gen. Rel. Grav., 1987, 19, 1213.
12. J.S. Jeavons, C.B.G. Melntosh and D.K. Sen, J. Math. Phys., 1975, 16, 320.
13. Zel'dovich, B. Ya., Mon. Not. R. Astron. Soc., 1972, 160.
14. T.W.B. Kibble, J. Phys., 1976, A9, 1387.
15. A.E. Everett, Phys. Rev., 1981, 4, 858.

16. A. Vilenkin, Phys. Rev. 1981, D24, 2082.
17. K.D. Krori, T. Chaudhury and C.R. Mahanta, Gen. Relativ. Gravit., 1994, 26, 265.
18. T. Singh and G.P. Singh, J. Math.Phys., 1991, 32, 2456.
19. T. Singh and G.P. Singh, Fortschr.Phys., 1993, 41, 737.
20. G.P. Singh, R.V. Deshpande and T. Singh, Pramana J. Phys., 2004, 63, 937.
21. F. Rahman, Chakravorty, S. Das, and J. Bera, Int. J.Mod. Phys., 2002, D 10, 1501.
22. F. Rahman, S. Chakravorty, N. Begum, M. Hossain and M. Kalal, Fizika B, 2002, 11, 57.
23. F. Rahman, S. Chakravorty, S. Das, M. Hossain, J. and Bera, Pramana J. Phys., 2003a, 60,453.
24. F. Rahman, G. Bag, N. Begum and S. Das, Bulg. J. Phys., 2005, 32, 10.
25. F. Rahman, S. Das, N. Begum and M. Hossain, Pramana J. Phys., 2003,61,153
26. G. Mohanty, K.L. Mahanta and R.R. Sahoo, Astrophys.Space Sci., 2006, 306, 269.
27. G. Mohanty, K.L. Mahanta, and B.K. Bishi, Astrophys.Space Sci., 2007, 310, 273.
28. F. Rahaman and J. Bera, Int.J.Mod.Phys., 2001, D 10,729.
29. S.B. Kalyanshetti and B.B. Waghmode, Gen. Relativ. Gravit., 1982, 14, 823.
30. D.R.K. Reddy and M.V.S. Rao, Astrophys.Space Sci., 2006, 302, 157.
31. D.R.K. Reddy and M.V.S. Rao, Astrophys.Space Sci., 2006, 305, 183.
32. R. Bali and N.K. Chandnani, Astrophys.Space Sci., 2009, 318, 225.
33. R. Bali and N.K. Chandnani, J. Math. Phys., 2008, 49, 032502.
34. S. Chakraborty and S. Ghosh, J. Mod. Phys., 2000, 9, 543.
35. A. Banarjee, S.B. Dutta Chaudhurary and A.K. Sanyal, J. Math. Phys. 1985, 26, 3010.
36. A. Pradhan, V.K. Yadav and I. Chakraborty, Int. J. Theor.Phys. 2001, 10, 339.
37. A. Pradhan and A.K. Vishwakarma, J. Geometry and Phys., 2004, 49, 32.
38. A. Pradhan and H. Amirhashchi, Astrophys. SpaceSci., 2011, 332, 441.
39. R. Bali and R. Tickekar, Chin. Phys.Lett. 2007, 24, 3290.
40. R. Bali and M. Kumawat, Int. J. Theor.Phys. 2009, 48, 3410.
41. R. Bali and Anjali, Astrophys.Space Sci., 2006, 302, 201.
42. R. Bali and U.K. Pareek and A. Pradhan, Chin. Phys. Lett., 2007, 24, 2455.
43. R. Tikekar and L.K. Patel, Pramana J. Phys., 1994, 42, 483.
44. X. Wang, Chin. Phys. Lett., 2006, 23, 1702.
45. S. Ram, M. Zeyauddin and C.P. Singh, J. Mod. Phys., 2008, 10, 339.
46. R. Venkateswarlu and K. Pavankumar, Astrophysics and Space Science. 2005, 298, 403.
47. G. Mohanty and G.C. Samanta, Int. Journal of theoretical physics, 2009, 48, 2311.
48. G.S. Rathore and K. Mandawat, Adv. Studies Theor. Phys., 2010, 4(5), 213-224
49. G.C. Samanta and D. Smrutirekha, Journal of Modern Physics, 2012, 3, 180-183.
50. P.S. Letelier, Phys. Rev., 1983, D28, 2414.

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