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Power Function of CST Procedure Based on two PTS in Four-Stage Unbalanced Nested Designs

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Abstract: In this paper we have been derived the expression for power and size of the CST procedure for four-stage partially balanced nested designs. Further, we have derived the formulae for power and size of the CST procedure for balance at third-stage as well as at fourth-stage within ith first-stage**.** The power of CST procedure denoted by $P_T(\theta)$ can be obtained by adding different power components P_{11}, P_{12}, P_{13} and P_{14}. i.e. $P_T(\theta) = P_{11} + P_{12} + P_{13} + P_{14}$.Where P_{11}, P_{12}, P_{13} and P_{14} be the probabilities of events E_{11}, E_{12}, E_{13} and E_{14} respectively.

Keywords: Power function**:** size, CST procedure, PTS, partially balanced nested designs.

INTRODUCTION

Size and power of test procedure incorporating preliminary tests of significance (PTS) have studied by Paull[1], Bozivich*et al*[2], Srivastava and Bozivich[3], Singh and Saxena[4] and Rao and Saxena[5]. Mead *et al*[6] studied empirically the size and power of the sometimes pooled test proposed by Bozivich*et al*[2].Brar[7] has studied theoretically the power function of conditionally specified test procedures, mean values and mean square error of the variance estimate for three-stage unbalanced nested designs. Test of significance is determined by specification of the population from which experimental data are sampled.

When the interaction effect term is doubtful we may test the hypothesis that the interaction effect is zero before we analyze the data. When hypothesis is accepted the effect term is eliminated from the model, such specification is termed as conditional specification and the objective test used in resolving the uncertainty of the parent model is known as preliminary test of significance (PTS). The procedure of testing a main hypothesis involving preliminary test is called conditionally specified test (CST) procedure. We consider the linear model for four-stage unbalanced design as

$$Y_{ijkl} = \mu + a_i + b_{ij} + c_{ijk} + e_{ijkl} \qquad \dots (1)$$

All terms in right hand side of the model except μ are assumed to be independent and normally distributed random variables with zero means and variances $\sigma_a^2, \sigma_b^2, \sigma_c^2$ and σ_e^2 respectively.

Where $Q_1 = \left[I - \sum_i^a \sum_j^{b_i} \sum_k^{c_{ij}} \frac{J_{n_{ijk}}}{n_{ijk}} \right], Q_2 = \left[\sum_i^a \sum_j^{b_i} \sum_k^{c_{ij}} \frac{J_{n_{ijk}}}{n_{ijk}} - \sum_i^a \sum_j^{b_i} \frac{J_{n_{ij}}}{n_{ij}} \right], Q_3 = \left[\sum_i^a \sum_j^{b_i} \frac{J_{n_{ij0}}}{n_{ij0}} - \sum_i^a \frac{J_{n_i}}{n_i} \right]$

$$Q_4 = \left[\sum_i^a \frac{J_{n_i}}{n_i} - \frac{J_n}{n} \right],$$

$$\tau_0 = \frac{(N - k_6)}{(c - b)}, \quad \tau_1 = \frac{(k_6 - k_5)}{(b - a)}, \tau_2 = \frac{(N - k_4)}{(b - a)},$$

$$\tau_3 = \frac{(k_5 - k_3)}{(a - 1)}, \tau_4 = \frac{(k_4 - k_2)}{(a - 1)}, \tau_5 = \frac{(N - k_1)}{(a - 1)}.$$

Where J_n denote an (n x n) matrix of unit elements and $\sum_i^k A_i$, the direct sum of matrices $A_1, A_2, \dots\dots, A_k$ as in Searle [8] and

$$b = \sum_i^a b_i, \; c = \sum_i^a \sum_j^{b_i} c_{ij}, \; N = \sum_i^a \sum_j^{b_i} \sum_k^{c_{ij}} n_{ijk}, \; n_{i00} = \sum_j \sum_k n_{ijk}, \; n_{ij0} = \sum_k n_{ijk}.$$

The k's that define in the above relations are functions of n_{ijk} 's namely,

$$k_1 = \frac{\sum_i^a n_{i00}^2}{N}, k_2 = \frac{\sum_i^a \sum_j^{b_i} n_{ij0}^2}{N}, k_3 = \frac{\sum_i^a \sum_j^{b_i} \sum_k^{c_{ij}} n_{ijk}^2}{N},$$

$$k_4 = \sum_i^a \left[\sum_j^{b_i} \frac{n_{ij0}^2}{n_{i00}} \right], k_5 = \sum_i^a \left[\sum_j^{b_i} \sum_k^{c_{ij}} \frac{n_{ijk}^2}{n_{i00}} \right], k_6 = \sum_i^a \sum_j^{b_i} \left[\sum_k^{c_{ij}} \frac{n_{ijk}^2}{n_{ij0}} \right].$$

Where Y_{ijkl} is equal to the sum of general mean μ and independent normal variables a_i, b_{ij}, c_{ijk} and e_{ijkl}, the vector $\underline{Y}$ is distributed as $N(\mu, \underline{J}, M)$, where $\underline{J}$ denotes an all in one column vector. The variance-covariance matrix M can be expressed as

$$M = \sigma_e^2\, I + \sigma_c^2 \sum\sum\sum J_{n_{ijk}} + \sigma_b^2 \sum\sum J_{n_{ij}} + \sigma_a^2 \sum J_{n_i} \qquad \ldots (2)$$

For testing the hypothesis $H_0 : \sigma_a^2 = 0$ against the alternative hypothesis $H_1 : \sigma_a^2 > 0$, an exact test of the hypothesis $H_0 : \sigma_a^2 = 0$ is not possible in general because $\tau_1 \neq \tau_3$ and $\tau_4 \neq \tau_2$. When $n_{ijk} = n$ and $c_{ij} = k$ for all i, j and k, an exact test is available because under this condition $\tau_1 = \tau_3$. Also from ANOVA **Table-1** an exact test for testing H_0 exists if $\sigma_b^2 = 0$ and $\sigma_c^2 = 0$. But we do not know whether σ_b^2 and σ_c^2 are zero or not.

Table-1: Analysis of variance table for four-stage unbalanced nested designs.

Sources of variation	**Degrees of freedom**	**Mean squares**	**E(MSS)**
Between A classes	$\eta_4 = a - 1$	$Y'Q_4Y = M_4$	$\sigma_e^2 + \tau_3\sigma_c^2 + \tau_4\sigma_b^2 + \tau_5\sigma_a^2$
Between B classes within A classes	$\eta_3 = \sum_i^a b_i - a$	$Y'Q_3Y = M_3$	$\sigma_e^2 + \tau_1\sigma_c^2 + \tau_2\sigma_b^2$
Between C classes within B classes	$\eta_2 = \sum_i^a \sum_j^{b_i} c_{ij} - \sum_i^a b_i$	$Y'Q_2Y = M_2$	$\sigma_e^2 + \tau_0\sigma_c^2$
Within C classes	$\eta_1 = N - \sum_i^a \sum_j^{b_i} c_{ij}$	$Y'Q_1Y = M_1$	σ_e^2

Under such condition, model (1) assumes the following forms:

$$Y_{ijkl} = \begin{cases} \mu + a_i + b_{ij} + c_{ijk} + e_{ijkl} & \text{for } \sigma_b^2 > 0, \sigma_c^2 > 0 \quad (3) \\ \mu + a_i + b_{ij} + e_{ijkl} & \text{for } \sigma_b^2 > 0, \sigma_c^2 = 0 \quad (4) \\ \mu + a_i + c_{ijk} + e_{ijkl} & \text{for } \sigma_b^2 = 0, \sigma_c^2 > 0 \quad (5) \\ \mu + a_i + e_{ijkl} & \text{for } \sigma_b^2 = 0, \sigma_c^2 = 0 \quad (6) \end{cases}$$

If $\sigma_b^2 \geq 0$, $\sigma_c^2 \geq 0$, the model (1) is known as conditionally specified model, however, if it is known with certainty that $\sigma_b^2 > 0, \sigma_c^2 > 0$ then appropriate model is (3) and which is known as unconditionally specified full model and if $\sigma_b^2 > 0, \sigma_c^2 = 0$ or $\sigma_b^2 = 0, \sigma_c^2 > 0$ or $\sigma_b^2 = 0, \sigma_c^2 = 0$, then the appropriate model are (4), (5) and (6) respectively which are also known as unconditionally specified model. If we assume the unconditionally specified full model given in (3) then $\sigma_b^2 > 0$ and $\sigma_c^2 > 0$ and we obtain ANOVA **Table-1**.

Then for testing H_0, the test statistic is $F_1 = \frac{M_4}{M'}$ where mean square $M_4 = \sigma_e^2 + \tau_3\sigma_c^2 + \tau_4\sigma_b^2$ under $H_0 : \sigma_a^2 = 0$ and $M' = \hat{\sigma}_e^2 + \tau_3\hat{\sigma}_c^2 + \tau_4\hat{\sigma}_b^2$

To obtain estimates of the variance components is accomplished by equating expected and observed mean squares in the ANOVA **Table-1** as follows:

$$\hat{\sigma}_e^2 = M_1 \quad \hat{\sigma}_c^2 = \frac{(M_2 - M_1)}{\tau_0} \text{ and } \hat{\sigma}_b^2 = \frac{\left[M_3 - M_1 - \frac{\tau_1}{\tau_0}(M_2 - M_1)\right]}{\tau_2}$$

Putting this value in M' we get

$$M' = \left[M_1\left(1 - \frac{\tau_3}{\tau_0} - \frac{\tau_4}{\tau_2} + \frac{\tau_1\tau_4}{\tau_0\tau_2}\right) + M_2\left(\frac{\tau_3}{\tau_0} - \frac{\tau_1}{\tau_0}\frac{\tau_4}{\tau_2}\right) + M_3\left(\frac{\tau_4}{\tau_2}\right)\right]$$

Now, taking $\frac{\tau_1}{\tau_0} = C_1$, $\frac{\tau_3}{\tau_0} = C_2$, $\frac{\tau_4}{\tau_2} = C_3$

For testing the null hypothesis $H_0 : \sigma_a^2 = 0$ against $H_1 : \sigma_a^2 > 0$, we have derived following CST procedure.

Reject H_0, if any of the four mutually exclusive events occurs in each of the following CST procedures:

CST Procedure

When $C_1 < 1,\ C_2 < 1,\ C_3 > 1,\ (1 - C_2 - C_3 + C_1 C_3) < 0,\ (C_2 - C_1 C_3) < 0$

$$p_{11} : \left\{\frac{M_2}{M_1} < \beta_1,\ \frac{M_3}{M_{12}} < \beta_2,\ \frac{M_4}{M_{123}} \ge \beta_3\right\}$$

$$p_{12} : \left\{\begin{array}{l} \frac{M_2}{M_1} \ge \beta_1,\ \frac{M_3}{[C_1 M_2 + (1 - C_1) M_1]} \ge \beta_4, \\ \frac{M_4}{[C_3 M_3 - (C_1 C_3 - C_2) M_2 - (C_2 + C_3 - 1 - C_1 C_3) M_1]} \ge \beta_5 \end{array}\right\}$$

$$p_{13}:\left\{\frac{M_2}{M_1}<\beta_1,\ \frac{M_3}{M_{12}}\geq\beta_2,\ \frac{M_4}{\left[C_3\,M_3-(C_3-1)M_2\right]}\geq\beta_6\right\}$$

$$p_{14}:\left\{\frac{M_2}{M_1}\geq\beta_1,\ \frac{M_3}{\left[C_1\,M_2+(1-C_1)\,M_1\right]}<\beta_4,\frac{M_4}{\left[C_2\,M_2+(1-C_2)\,M_1\right]}\geq\beta_7\right\} \qquad \ldots (7)$$

If α and β are the probabilities of type I and type II errors respectively, the quantity $(1-\beta)$ called the power of the test and α the size of the test. The power of the test depends upon the difference between the parameter value specified by H_0 and actual value of the parameter. Power function $(1-\beta)$ can be expressed as a function of true parameter say θ.

If T is a test of H_0 the power function of T is defined as the probability of rejection of H_0, when the distribution from which the sample is drawn is parameterized by θ and denoted by $P_T(\theta)$. Let P_{11}, P_{12}, P_{13} and P_{14} be the probabilities of events E_{11}, E_{12}, E_{13} and E_{14} respectively. Since E_{11}, E_{12}, E_{13} and E_{14} are mutually exclusive, the power function of the CST procedure is the probability $P_T(\theta)$ which is given by

$$P_T(\theta)=\sum_{i=1}^{4}\Pr\left(E_{1i}\middle|H_1\right)=\sum_{i=1}^{4}P_{1i} \qquad \ldots (8)$$

Where $P_{1i}=\Pr\left(E_{1i}\middle|H_1\right)$ and $H_1:\sigma_a^2>0$. Thus, the size of the CST procedure will be

$\sum_{i=1}^{4}\Pr\left(E_{1i}\middle|H_0\right)$, where $H_0:\sigma_a^2=0$.

Integral expressions for power components P_{11}, P_{12}, P_{13} and P_{14} when $\eta_1, \upsilon_2, \upsilon_3$ and υ_4 are Even Numbers: We know that $\eta_1 M_1$ is distributed as $\sigma_1^2\chi_{\eta_1}^2$ where $\chi_{\eta_1}^2$ is the central chi-square statistic based on η_1 degrees of freedom. According to Box loc. cit., the sum of squares $\eta_2 M_2$, $\eta_3 M_3$ and $\eta_4 M_4$ are approximately distributed as $d_2\chi_{\upsilon_2}^2$, $d_3\chi_{\upsilon_3}^2$ and $d_4\chi_{\upsilon_4}^2$, where $\chi_{\upsilon_2}^2$, $\chi_{\upsilon_3}^2$ and $\chi_{\upsilon_4}^2$ are the central chi-square statistics based on υ_2, υ_3, and υ_4 degrees of freedom respectively and

$d_2, d_3, d_4, \upsilon_2, \upsilon_3$ and υ_4 the sum of squares $\eta_i M_i$ (i = 1, 2, 3, 4) are distributed independently, the joint p.d.f. of M_i (i=1, 2, 3, 4) can be written as

$$g(M_1, M_2, M_3, M_4) = C^* M_1^{\frac{\eta_1}{2}-1} \cdot M_2^{\frac{\upsilon_2}{2}-1} \cdot M_3^{\frac{\upsilon_3}{2}-1} \cdot M_4^{\frac{\upsilon_4}{2}-1}$$
$$\exp\left\{-\frac{1}{2}\left(\frac{\eta_1 M_1}{\sigma_1^2} + \frac{\eta_2 M_2}{d_2} + \frac{\eta_3 M_3}{d_3} + \frac{\eta_4 M_4}{d_4}\right)\right\} \qquad \ldots(9)$$

Where C^* is the constant and is independent of M_i's. Applying the transformations

$$u = \frac{\eta_2 M_2}{\eta_1 M_1 \theta_{21}}, \quad v = \frac{\eta_3 M_3}{\eta_2 M_2 \theta_{32}}, \quad w = \frac{\eta_4 M_4}{\eta_3 M_3 \theta_{43}}, \quad x = \frac{\eta_1 M_1}{\eta_4} \qquad \ldots (10)$$

Where $\theta_{21} = \frac{d_2}{\sigma_1^2}$, $\theta_{32} = \frac{d_3}{d_2}$, $\theta_{43} = \frac{d_4}{d_3}$ Then, the joint p.d.f. of u, v, w and x is given by

$$h(u, v, w, x) = C_1^* x^{\frac{\eta_{1234}}{2}-1} u^{\frac{\eta_{234}}{2}-1} v^{\frac{\eta_{34}}{2}-1} w^{\frac{\upsilon_4}{2}-1} \exp\left\{-\left(\frac{\eta_4 x}{2\sigma_1^2}\right)(1+u+uv+uvw)\right\} \qquad \ldots (11)$$

Where $\eta_{1234} = \eta_1 + \upsilon_2 + \upsilon_3 + \upsilon_4$, $\eta_{234} = \upsilon_2 + \upsilon_3 + \upsilon_4$, $\eta_{34} = \upsilon_3 + \upsilon_4$ and

$$C_1^* = \frac{\left(\frac{\eta_4}{2\sigma_1^2}\right)^{\frac{\eta_{1234}}{2}}}{\Gamma\left(\frac{\eta_1}{2}\right)\Gamma\left(\frac{\upsilon_2}{2}\right)\Gamma\left(\frac{\upsilon_3}{2}\right)\Gamma\left(\frac{\upsilon_4}{2}\right)} \qquad \ldots (12)$$

Integrating out x over the range 0 to∞, we get the joint p.d.f. of u, v and w as

$$f(u, v, w) = \frac{C_2^* u^{\frac{\eta_{234}}{2}-1} v^{\frac{\eta_{34}}{2}-1} w^{\frac{\upsilon_4}{2}-1}}{(1+u+uv+uvw)^{\frac{\eta_{1234}}{2}}} \qquad \ldots (13)$$

Where $C_2^* = \frac{\Gamma\left(\frac{\eta_{1234}}{2}\right)}{\Gamma\left(\frac{\eta_1}{2}\right)\Gamma\left(\frac{\eta_2}{2}\right)\Gamma\left(\frac{\eta_3}{2}\right)\Gamma\left(\frac{\eta_4}{2}\right)}$.

The limits of integration of the new variables u, v and w corresponding to different components are as follows:

For P_{11} : $0 \le u \le a_1, \quad 0 \le v \le \dfrac{b_1\left(1+u\theta_{21}\right)}{u}, \quad \dfrac{e\left(1+u\theta_{21} + uv\theta_{21}\theta_{32}\right)}{uv} \le w < \infty,$

For P_{12} : $a_1 \le u < \infty, \ \dfrac{b_2\left(\eta_1 C_1 u\theta_{21} + (1-C_1)\upsilon_2\right)}{u} \le v < \infty$

$$\frac{e_1\left(uv\theta_{21}\theta_{32}\eta_1\upsilon_2 C_3 - (C_1C_3 - C_2)u\theta_{21}\eta_1\upsilon_3 - (C_2 + C_3 - 1 - C_1C_3)\upsilon_2\upsilon_3\right)}{uv} \le w < \infty$$

For P_{13}: $0 \le u \le a_1, \ \dfrac{b_1\left(1+u\theta_{21}\right)}{u} \le v \le \infty, \ \dfrac{e_2\left(vC_3\upsilon_2\theta_{32} - (C_3-1)\upsilon_3\right)}{v} \le w < \infty$

For P_{14}: $a_1 \le u < \infty, \ 0 \le v < \dfrac{b_2\left(\eta_1 C_1 u\theta_{21} + (1-C_1)\upsilon_2\right)}{u},$

$$\frac{e_3\left(uC_2\eta_1\theta_{21} + (1-C_2)\upsilon_2\right)}{uv} \le w < \infty$$

Where $a_1 = \dfrac{\upsilon_2\beta_1}{\eta_1\theta_{21}}, \ b_1 = \dfrac{\upsilon_3\beta_2}{\eta_{12}\theta_{21}\theta_{32}}, \ e = \dfrac{\upsilon_4\beta_3}{\eta_{123}\theta_{21}\theta_{32}\theta_{43}}, \ b_2 = \dfrac{\upsilon_3\beta_4}{\eta_1\upsilon_2\theta_{21}\theta_{32}},$

$$e_1 = \frac{\upsilon_4\beta_5}{\eta_1\upsilon_2\upsilon_3\theta_{21}\theta_{32}\theta_{43}}, \ e_2 = \frac{\upsilon_4\beta_6}{\upsilon_2\upsilon_3\theta_{32}\theta_{43}}, \ e_3 = \frac{\upsilon_4\beta_7}{\eta_1\upsilon_2\theta_{21}\theta_{32}\theta_{43}} \quad \ldots (14)$$

The integral expression for the four components may therefore, be written as follows

$$P_{11} = C_2^* \int_0^{a_1} \int_0^{b_1(1+u\theta_{21})/u} \int_{e(1+u\theta_{21}+uv\theta_{21}\theta_{32})/uv}^{\infty} f(u,v,w)\,dw\,dv\,du \quad \ldots (15)$$

$$P_{12} = C_2^* \int_{a_1}^{\infty} \int_{b_2(\eta_1 C_1\theta_{21}+u+(1-C_1)\upsilon_2)/u}^{\infty} \int_{e_1(uv\theta_{21}\theta_{32}\eta_1\upsilon_2 C_3-(C_1C_3-C_2)u\theta_{21}\eta_1\upsilon_3-(C_2+C_3-1-C_1C_3)\upsilon_2\upsilon_3)/uv}^{\infty} f(u,v,w)\,dw\,dv\,du \quad \ldots (16)$$

$$P_{13} = C_2^* \int_0^{a_1} \int_{b_1(1+u\theta_{21})/u}^{\infty} \int_{e_2\left(v\theta_{32}\,\upsilon_2 C_3-(C_3-1)\upsilon_3\right)/v}^{\infty} f(u,v,w)\,dw\;dv\,du \qquad \ldots (17)$$

$$P_{14} = C_2^* \int_{a_1}^{\infty} \int_0^{b_2\left(\eta_1 C_1\theta_{21}u+(1-C_1)\upsilon_2\right)/u} \int_{e_3\left(u\theta_{21}\,\eta_1 C_2+(1-C_2)\upsilon_2\right)/uv}^{\infty} f(u,v,w)\,dw\,dv\,du \qquad \ldots (18)$$

Where $f(u,v,w)$ is given by (13)

Series Formula for Power Component P_{11}:

Integrating out w from (15) by using the following standard result,

$$\int_{\beta}^{\infty} \frac{x^{m-1}dx}{(L_1+L_2x)^{m+n}} = \sum_{i=0}^{m-1} \frac{(-1)^i\left({}^{m-1}c_i\right)(L_1)^i}{(n+i)(L_2)^m(L_1+L_2\beta)^{n+i}} \qquad \ldots (19)$$

Where ${}^m c_n = \dfrac{\Gamma m}{\Gamma n\;\Gamma(m-n)}$ Then, we obtain P_{11} as

$$P_{11} = C_2^*\,S_i \int_0^{a_1}\int_0^{b_1 1+u\theta_{21}/u} \frac{u^{\frac{\eta_{23}}{2}-1}\,v^{\frac{\upsilon_3}{2}-1}(1+u+uv)^i\,dv\,du}{\left(1+e+u\,(1+e\theta_{21})+uv(1+e\theta_{21}\theta_{32})\right)^{\frac{\eta_{123}}{2}+i}} \qquad \ldots (20)$$

$$\text{Where } S_i = \frac{\sum_{i=0}^{\frac{\upsilon_4}{2}-1}\left({}^{\frac{\upsilon_4}{2}-1}c_i\right)(-1)^i}{\left(\frac{\eta_{123}}{2}+i\right)} \qquad \ldots (21)$$

Integrating out v from (20) by using the following result,

$$\int_0^{\beta} \frac{x^{m-1}(a+bx)^p\,dx}{(L_1+L_2x)^{m+n+p}} = \frac{\sum_{q=0}^{p}\left({}^p c_q\right)a^q\,b^{p-q}\sum_{r=0}^{m+p-q-1}\left({}^{m+p-q-1}c_r\right)(-1)^r}{(L_2)^{m+p-q}\;(n+q+r)} \times \left[\frac{1}{(L_1)^{n+q}} - \frac{(L_1)^r}{(L_1+L_2\beta)^{n+q+r}}\right] \qquad \ldots (22)$$

$$P_{11}=\frac{C_2^{*}\,S_{ij}}{(D_1)^{\frac{\upsilon_3}{2}+i-j}}\int_0^{a_1}u^{\frac{\upsilon_2}{2}-1}(1+u)^j\times\left[\frac{1}{\left(1+e+u(1+e\theta_{21})\right)^{\frac{\eta_{12}}{2}+j}}\right.$$

$$\left.-\frac{\left(1+e+u(1+e\theta_{21})\right)^r}{\left(1+e+(1+e\theta_{21}\theta_{32})b_1+u\left(1+e\theta_{21}+b_1\theta_{21}(1+e\theta_{21}\theta_{32})\right)\right)^{\frac{\eta_{12}}{2}+j+r}}\right]du \qquad \ldots(23)$$

$$\text{Where } S_{ij}=S_i\;\frac{\sum_{j=0}^{i}\left({}^{i}c_j\right)\sum_{r=0}^{\frac{\upsilon_3}{2}+i-j-1}\left({}^{\frac{\upsilon_3}{2}+i-j-1}c_r\right)(-1)^r}{\left(\frac{\eta_{12}}{2}+j+r\right)} \qquad \ldots(24)$$

Using the binomial expansion for $(1+u)^j$ and integrating out u with the help of following standard results

$$\int_0^{\beta}\frac{x^{m-1}}{(a+bx)^{m+n}}dx=\frac{B_{c_0}(m,n)}{a^n\,b^m} \qquad \ldots(25)$$

$$\int_0^{\beta}\frac{x^{m-1}(a+bx)^p}{(L_1+L_2x)^{m+n+p}}dx=\sum_{q=0}^{p}\left({}^{p}c_q\right)b^q\,a^{p-q}\,\frac{B_{c_0}(m+q\,,n+p-q)}{(L_1)^{n+p-q}(L_2)^{m+q}} \qquad \ldots(26)$$

Where $c_0=\dfrac{L_2\beta}{L_1+L_2\beta}$ Then, we obtain P_{11} as

$$P_{11}=\frac{C_2^{*}}{(D_1)^{\frac{\upsilon_3}{2}+i-j}}\left[S_{ijk}\left(\frac{B_{x_0}\left(\frac{\upsilon_2}{2}+j-k\,,\frac{\eta_1}{2}+k\right)}{(r_1)^{\frac{\eta_1}{2}+k}(r_2)^{\frac{\upsilon_2}{2}+j-k}}\right)-S_{ijkl}\left(\frac{(r_1)^{r-l}(r_2)^{l}B_{x_0'}\left(\frac{\upsilon_2}{2}+k+l,\frac{\eta_1}{2}+j+r-k-l\right)}{(r_1')^{\frac{\eta_1}{2}+j+r-k-l}(r_2')^{\frac{\upsilon_2}{2}+k+l}}\right)\right] \qquad \ldots(27)$$

Where

$$S_{ijk} = S_{ij} \sum_{k=0}^{j} \left({}^{j}c_{k} \right) \qquad \ldots (28)$$

And $$S_{ijkl} = S_{ijk}\, r \sum_{l=0}^{r} \left({}^{r}c_{l} \right) (-1)^{l} \qquad \ldots (29)$$

Also, $x_0 = \dfrac{r_2\, a_1}{(r_1 + r_2\, a_1\}}$, $x_0' = \dfrac{r_2'\, a_1}{(r_1' + r_2' a_1)}$, $D_1 = (1 + e\,\theta_{21}\theta_{32})$, $r_1 = (1+e)$,

$$r_2 = (1 + e\,\theta_{21})\ ,\ r_1' = r_1 + b_1(1 + e\,\theta_{21}\,\theta_{32})\ ,\ r_2' = r_2 + b_1 (1 + e\,\theta_{21}\,\theta_{32})\,\theta_{21} \qquad \ldots (30)$$

Series Formula for Power Component P_{12}

Integrating out w from (16) with the help of integral given by (19), we get

$$P_{12} = C_2^{*}\ S_i \int_{a_1}^{\infty} \int_{b_2\left(\eta_1 C_1 \theta_{21} u + (1-C_1)\upsilon_2\right)/u}^{\infty} u^{\frac{\eta_{23}}{2}-1}\, v^{\frac{\upsilon_3}{2}-1} (1 + u + uv)^{i}\, dv\, du$$

$$P_{12} = \frac{C_2^{*}\, S_{ijkl}}{(D_2)^{\frac{\upsilon_3}{2}+i-j}} \left[\frac{(l_2)^{r-l}\, B\left(\frac{\upsilon_2}{2} + j + r - k - l\ ,\ \frac{\eta_1}{2} + k + l \right)}{(l_1')^{\frac{\eta_1}{2}}\ (l_2')^{\frac{\upsilon_2}{2}+j+r-k-l}} - \frac{(l_2)^{l}\, (l_1)^{r-l}\, B_{x_1}\left(\frac{\upsilon_2}{2} + k + l\ ,\ \frac{\eta_1}{2} + j + r - k - l \right)}{(l_1')^{\frac{\eta_1}{2}+j+r-k-l}\ (l_2')^{\frac{\upsilon_2}{2}+k+l}} \right] \qquad \ldots (31)$$

Where S_{ijkl} is defined in (29). Also,

$$x_1 = \frac{l_2'\, a_1}{(l_1' + l_2' a_1)}\ ,\quad D_2 = \left(1 + e_1\,\theta_{21}\,\theta_{32}\ \eta_1\,\upsilon_2\ C_3\right)\ ,\quad l_1 = \left(1 - (C_2 + C_3 - 1 - C_1 C_3)\,\upsilon_2\,\upsilon_3\, e_1\right)\ ,$$

$$l_2 = \left(1 - (C_1 C_3 - C_2)\, e_1\, \eta_1\, \upsilon_3\, \theta_{21}\right),\ l_1' = \left(l_1 + b_2\,(1 - C_1)\ \upsilon_2\,(1 + e_1\,\theta_{21}\,\theta_{32}\,\eta_1 \upsilon_2\, C_3)\right),$$

$$l_2' = \left(l_2 + b_2\,(1 + e_1\,\theta_{21}\,\theta_{32}\ \eta_1\,\upsilon_2\, C_3)\,\eta_1\,\theta_{21}\right) \qquad \ldots (32)$$

Series Formula for Power Component P_{13}

Integrating out w from (17) with the help of integral given by (19)

$$P_{13} = C_2^* S_i \int_0^{a_1} \int_{b_1(1+u\theta_{21})/u}^{\infty} \frac{u^{\frac{\eta_{23}}{2}-1} v^{\frac{\upsilon_3}{2}-1} (1+u+uv)^i \, dv\, du}{\left[1+u\left(1-e_2(C_3-1)\upsilon_3\right)+uv\left(1+e_2 C_3 \upsilon_2 \theta_{32}\right)\right]^{\frac{\eta_{123}}{2}+i}}$$

We get

$$P_{13} = \frac{C_2^* S_{ijkl}}{(D_3)^{\frac{\upsilon_3}{2}+i-j}} \left[\frac{(t_1)^{r-l} (t_2)^l B_{x_2}\left(\frac{\upsilon_2}{2}+k+l, \frac{\eta_1}{2}+j+r-k-l\right)}{(t_1')^{\frac{\eta_1}{2}+j+r-k-l} (t_2')^{\frac{\upsilon_2}{2}+k+l}} \right] \quad \ldots (33)$$

Where S_{ijkl} is defined in (29).Also,

$$x_2 = \frac{t_2' a_1}{(t_1' + t_2' a_1)}, D_3 = \left(1+e_2 C_3 \upsilon_2 \theta_{32}\right), t_1 = 1, t_2 = \left(1+e_2(1-C_3)\upsilon_3\right)$$

$$t_1' = \left(t_1 + b_1(1+e_2 C_3 \theta_{32} \upsilon_2)\right),\; t_2' = \left(t_2 + b_1 \theta_{21}(1+e_2 C_3 \theta_{32} \upsilon_2)\right) \quad \ldots (34)$$

Series Formula for Power Component P_{14}

Integrating out w from (18) by using the integral given by (19) we get;

$$P_{14} = C_2^* S_i \times \int_{a_1}^{\infty} \int_0^{b_2(\eta_1 C_1 \theta_{21} u + (1-C_1)\upsilon_2)/u} u^{\frac{\eta_{23}}{2}-1} v^{\frac{\upsilon_3}{2}-1} (1+u+uv)^i \, dv\, du$$

$$\frac{1}{\left[\left(1+e_3(1-C_2)\upsilon_2\right)+u\left(1+\eta_1 C_2 e_3 \theta_{21}\right)+uv\right]^{\frac{\eta_{123}}{2}+i}}$$

$$P_{14} = C_2^* \left[S_{ijk} \left(\frac{B\left(\frac{\upsilon_2}{2}+j-k, \frac{\eta_1}{2}+k\right) - B_{x_3}\left(\frac{\upsilon_2}{2}+j-k, \frac{\eta_1}{2}+k\right)}{(s_1)^{\frac{\eta_1}{2}+k} (s_2)^{\frac{\upsilon_2}{2}+j-k}} \right) \right.$$

$$-S_{ijkl}\left(\frac{(s_2)^{r-l}\,B\left(\frac{\upsilon_2}{2}+j+r-k-l\ ,\ \frac{\eta_1}{2}+k+l\right)}{(s_1')^{\frac{\eta_1}{2}}\,(s_2')^{\frac{\upsilon_2}{2}+j+r-k-l}}\right.$$

$$\left.\left.-\frac{(s_1)^{r-l}(s_2)^{l}\,B_{x_3'}\left(\frac{\upsilon_2}{2}+k+l\ ,\ \frac{\eta_1}{2}+j+r-k-l\right)}{(s_1')^{\frac{\eta_1}{2}+j+r-k-l}\,(s_2')^{\frac{\upsilon_2}{2}+k+l}}\right)\right] \qquad \ldots (35)$$

Where S_{ijk} and S_{ijkl} is defined in (28) and (29) respectively.

$$x_3=\frac{s_2\,a_1}{(s_1+s_2\,a_1)}\ ,\quad x_3'=\frac{s_2'\,a_1}{(s_1'+s_2'\,a_1)}\ ,\quad s_1=\left(1+e_3(1-C_2)\upsilon_2\right)\ ,\quad s_2=\left(1+e_3\,C_2\,\eta_1\,\theta_{21}\right)\ ,$$
$$s_1'=\left(s_1+b_2(1-C_1)\upsilon_2\right),\ s_2'=\left(s_2+b_2\,C_1\,\eta_1\,\theta_{21}\right) \qquad \ldots (36)$$

$$A_{i3}=\frac{\sum_{i=0}^{\frac{\upsilon_3}{2}-1}\binom{\frac{\upsilon_3}{2}-1}{c_i}(-1)^i}{\left(\frac{\eta_{12}}{2}+3+i\right)\left(\frac{\eta_{123}}{2}+3\right)},\ A_{i3}'=\frac{\sum_{i=0}^{\frac{\upsilon_3}{2}}\binom{\frac{\upsilon_3}{2}}{c_i}(-1)^i}{\left(\frac{\eta_{12}}{2}+2+i\right)\left(\frac{\eta_{123}}{2}+3\right)},$$

$$A_{i3}''=\frac{\sum_{i=0}^{\frac{\upsilon_3}{2}+1}\binom{\frac{\upsilon_3}{2}+1}{c_i}(-1)^i}{\left(\frac{\eta_{12}}{2}+1+i\right)\left(\frac{\eta_{123}}{2}+3\right)},\ A_{i3}'''=\frac{\sum_{i=0}^{\frac{\upsilon_3}{2}+2}\binom{\frac{\upsilon_3}{2}+2}{c_i}(-1)^i}{\left(\frac{\eta_{12}}{2}+i\right)\left(\frac{\eta_{123}}{2}+3\right)}$$

RESULT

We obtained the power function of CST procedure denoted by $P_T(\theta)$ in (8) by adding the series formulae for power components P_{11}, P_{12}, P_{13} and P_{14} from equation (27), (31), (33) and (35) respectively in case when η_1, υ_2, υ_3 and υ_4 i.e. the degrees of freedom for various sources of variation are even numbers i.e. $P_T(\theta)=P_{11}+P_{12}+P_{13}+P_{14}$.

REFERENCES

1. Dangi. Inference for variance components in unbalanced nested design, unpublished Ph. D. thesis, M.L.S. University, Udaipur. 2006.
2. A.E. Paull: On a preliminary test for pooling mean squares in the analysis of variance. *Ann. Math. Stat*. 1950, **21**, 539-556.
3. C.V. Rao and K.P. Saxena: Some results on size in a mixed model using two preliminary tests of significance. *J. Ind. Soc. Agric. Stat*. 1980, **32**, 62-70.
4. H. Bozivich, T.A. Bancroft and H.O. Hartley: Power of analysis of variance test procedures for certain incompletely specified model-I. *Ann. Math. Stat*. 1956, **7**, 1017-1043.
5. L. Singh and K.P. Saxena: On power of sometime pooled test in a split plot design. *Sankhya-B*. 1980, **42**, 153-164.
6. M.A. Ali and S.R. Srivastava: On certain approximations of power of a test procedure using two preliminary tests in a mixed model. *Bull. Math Stat*. 1981, **19** (3), 103-113.
7. R. Mead, T.A. Bancroft and C.P. Han: Power of analysis of variance test procedures for incompletely specified fixed models. *Annals of Statistics*. 1975, **3**, 797-808.
8. R.L. Anderson and T.A. Bancroft: *Statistical theory in research.* Hill Book Company, New Yark. 1952.
9. S.R. Searle: *Linear models*. John Wiley, New York 1971.
10. S.R. Srivastava and H. Bozivich: Power of certain analysis of variance test procedures involving preliminary tests. The 33rd Session of International Statistical Institute, Paris. 1961.
11. T.A. Bancroft: On biases in estimation due to the use of preliminary tests of significance. *Ann. Math. Stat.* 1944, **15**, 190-204.
12. W.S. Brar. Estimation and testing of variance components in unbalanced designs. Unpublished Ph.D. Thesis, University of Rajasthan, Jaipur, India. 1990.

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