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## A Theoretical Study of Condensed Matter in Super Strong Magnetic field and an Evaluation of Cohesive Energies of Condensed Matter

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**Abstract:** We have evaluated cohesive energies of hydrogen, helium, carbon, oxygen, silicon and Iron matter in the presence of super strong magnetic field. Our theoretical results indicate that cohesive energies increase with increase of magnetic field strength B. The increase is small for smaller value of z and becomes larger and larger for higher Z values. Our theoretical results are in good agreement with other theoretical workers.

**Keywords:** Cohesive energies, super strong magnetic field, exchange energies, Adjustable parameters, neutron star surface

### INTRODUCTION

In this paper, we have discussed the method of evaluation of cohesive energies of Hydrogen, Carbon, Silicon and Iron matter in the presence of super strong magnetic field. In earlier papers<sup>1,2</sup>, we have presented the method of evaluation of binding energy with and without exchange energies. We have also evaluated the binding energies of Silicon and Iron matter with and without exchange energy term. We have compared our theoretical results with that of Müller<sup>3</sup>, Hillerbran and Müller<sup>4</sup> in the case of Helium, Carbon and Iron matter with different values of B. These evaluations are based on the theoretical

formalism of J.E. Skjervold and E. Østgaard<sup>5,6</sup>. Our theoretical result indicates that cohesive energy increases with increases of magnetic field strength  $B$ . The increase is small for smaller  $z$  value but becomes larger and larger for higher  $z$  values. It is also noticed that with the inclusion of exchange energy term in the calculation there is an enhancement of the values of the cohesive energy of all the above mentioned matter. However, as a function of magnetic field  $B$  the trend is same as with case of without exchange term.

**Mathematical formulae used in the evaluation:** We have taken the help of the expression used in Paper<sup>1,2</sup>.

The ground state energy for an atom in a super strong magnetic field when exchange terms are neglected is written as

$$E = -159.65 [B (10^{12}G)]^{2/5} Z^{9/5} \text{eV} \quad (1)$$

In Thomas–Fermi method, the ground state energy is written as

$$E = -153.47 [B (10^{12}G)]^{2/5} Z^{9/5} \text{eV} \quad (2)$$

When exchange energy term is included then ground state energy is given by

$$E = -2.475z^{19/6}z^{-1}[1.5+3^{-2}Z^{-2/3} \times (\ln+0.6279)] + 5.036^{-62}Z^4E_H \quad (3)$$

Where

$$E_H = e^2/2a_0 = 13.6 \text{ eV}$$

Here  $\eta$ ,  $\xi$  and  $z$  are the parameters. The details are given in paper I & II. The ground state energy for an atom in a super strong magnetic field, when exchange terms are included has been obtained also by Thomas–Fermi–Dirac method (Skjervold and Østgaard, 1984) and is given by

$$E = [-153.47 - 22.37 [B(10^{12}G)]^{-1/5} Z^{-2/5}] \times [B(10^{12}G)]^{2/5} Z^{9/5} \text{eV} \quad (4)$$

we have total energy (ground state energy) when the exchange term is neglected is written as

$$E = -\left(\frac{Z^2 e^2}{1}\right) \left[ \ln\left(\frac{2l}{R}\right) - (\epsilon - C_1) \right] + (z^3 \pi^2 \hbar^2 / 6ml^2)(\hat{\rho} / R)^4 \quad (5)$$

Minimizing with respect to  $l$  and  $R$  we have

$$\ln\left(\frac{2l}{R}\right) - (\epsilon - C_1 + 1) = (2z\pi^2 \hbar^2 / 6e^2 ml) \left(\frac{\hat{\rho}}{R}\right)^4$$

Which can be combined to give:

$$\ln(2l/R) = -C_1 + 3/2 \quad (6)$$

$$l = 2.87 R$$

The condition for  $\eta$  is given by

$$\eta = a_0 / (2^{1/2} Z^{3/2} \hat{\rho}) = 1.034 \times 10^{-5} (B/z)^{1/2}$$

$$\hat{\rho} = a_0 / (2^{1/2} Z^{3/2} \eta) \quad (7)$$

And

$$R = 1.08 a_0 z^{-1-4/5}$$

$$= 1.563 \pi^{2/5} a_0 \left( \frac{B_0}{B} \right)^{2/5} z^{1/5} \quad (8)$$

Where

$$B_0 = 1.17 \times 10^9 \text{G}$$

Using equations (5) and (6) and (8) we have evaluated the binding energies of silicon and Iron matter without exchange term included. The results are shown in **Table 1 and 2** respectively.

Similarly, using relation (3) we have determined the binding energies of silicon and Iron with exchange energy. The results are shown in **Table 3 and 4** respectively.

The cohesive energy of an atom in a matter when exchange term are ignored is

$$E_b = -153.47 + 159.65 [B (10^{12} \text{G})]^{2/5} Z^{9/6} \text{eV}$$

$$= 6.18 [B (10^{12} \text{G})]^{2/5} Z^{9/5} \text{eV} \quad \eta > 1 \quad (9)$$

The cohesive energy is the difference between the binding energies of free atoms and of atoms in condensed matter.

Now for hydrogen atom,  $z = 1$

$$E_b = 0.0062 [B (10^{12} \text{G})]^{2/5} \text{KeV} \quad B > 10^{10} \text{G} \quad (10)$$

For Helium,  $Z = 2$

$$E_b = 0.022 [B (10^{12} \text{G})]^{2/5} \text{KeV} \quad B > 10^{10} \text{G} \quad (11)$$

For Carbon,  $Z = 6$

$$E_b = 0.155 [B (10^{12} \text{G})]^{2/5} \text{KeV} \quad B > 10^{12} \text{G} \quad (12)$$

For Oxygen,  $Z = 8$

$$E_b = 0.0261 [B (10^{12} \text{G})]^{2/5} \text{KeV} \quad B > 10^{12} \text{G} \quad (13)$$

For Silicon,  $Z = 14$

$$E_b = 0.715 [B (10^{12} \text{G})]^{2/5} \text{KeV} \quad B > 10^{13} \text{G} \quad (14)$$

For Iron,  $Z = 26$

$$E_b = 2.177 [B (10^{12}G)]^{2/5} \text{ KeV} \quad B > 10^{14}G \quad (15)$$

**Table-1:** Dimensions and binding energies of Silicon matter in super strong magnetic field without exchange terms

B (10 <sup>12</sup> G)	η	R (a <sub>0</sub> )	l(a <sub>0</sub> )	–E(KeV)
1	6.20	0.281	0.807	18.5
5	0.44	0.148	0.424	35.1
10	0.62	0.112	0.321	46.4
50	1.40	0.059	0.169	88.3
100	1.97	0.045	0.128	116.5
500	4.41	0.023	0.067	211.7
600	4.98	0.022	0.060	238.6
700	5.20	0.021	0.058	249.6
800	5.68	0.020	0.055	272.8
900	5.96	0.019	0.053	284.5
1000	6.24	0.018	0.051	292.5

**Table- 2:** Dimensions and binding energies of Iron matter in super strong magnetic field without exchange terms

B (10 <sup>12</sup> G)	η	R(a <sub>0</sub> )	l(a <sub>0</sub> )	–E(KeV)
1	0.08	0.319	0.914	56
5	0.17	0.167	0.480	107
10	0.25	0.127	0.364	141
50	0.55	0.067	0.191	269
100	0.78	0.050	0.145	355
500	1.74	0.027	0.076	676
600	1.86	0.025	0.070	692
700	2.02	0.024	0.062	712
800	2.27	0.022	0.060	768
900	2.34	0.021	0.059	802
1000	2.47	0.020	0.058	891

**Table-3:** Evaluation of binding energy and exchange energy of Silicon matter in super strong magnetic field with exchange term

<b>B (10<sup>12</sup>G)</b>	<b><math>\eta</math></b>	<b><math>\xi</math></b>	<b>R (a<sub>0</sub>)</b>	<b><i>l</i>(a<sub>0</sub>)</b>	<b>–E<sub>ex</sub>(KeV)</b>	<b>–E(KeV)</b>
1	0.20	1.62	0.267	0.766	1.45	20.0
5	0.44	1.91	0.141	0.404	2.27	37.4
10	0.62	2.06	0.107	0.307	2.73	49.1
50	1.40	2.43	0.057	0.164	4.2	92.5
100	1.97	2.61	0.043	0.123	5.0	121.5
500	4.41	3.09	0.023	0.066	7.5	229.2
600	4.98	3.15	0.020	0.063	7.9	247.5
700	5.20	3.20	0.0190	0.060	8.5	259.8
800	5.68	3.25	0.0182	0.058	8.7	267.5
900	5.96	3.30	0.0179	0.055	8.9	286.8
1000	6.24	3.32	0.012	0.049	9.0	301.5

**Table-4:** Evaluation of binding energy and exchange energy of Iron matter in super strong magnetic field with exchange term

<b>B (10<sup>12</sup>G)</b>	<b><math>\eta</math></b>	<b><math>\xi</math></b>	<b>R (a<sub>0</sub>)</b>	<b><i>l</i>(a<sub>0</sub>)</b>	<b>–E<sub>ex</sub>(KeV)</b>	<b>–E(KeV)</b>
1	0.08	1.51	0.298	0.856	3.1	59
5	0.17	1.78	0.165	0.475	5.0	112
10	0.25	1.92	0.121	0.348	6.0	147
50	0.55	2.26	0.065	0.186	9.3	278
100	0.78	2.43	0.049	0.141	11.2	366
500	1.74	2.86	0.026	0.075	17.0	693
600	1.86	2.98	0.025	0.073	18.2	700
700	2.02	3.02	0.024	0.070	18.9	728
800	2.27	3.05	0.023	0.067	19.2	768
900	2.34	3.06	0.022	0.060	19.8	859
1000	2.47	3.0	0.020	0.057	20.3	911

The cohesive energy  $-E_b$  (keV) for atoms in matter in super strong magnetic fields with exchange neglected and exchange terms included are shown in **Tables 5 and 6** respectively. The comparison of

binding energies  $E$ , exchange energy  $E_{\text{ex}}$  and Cohesive energy  $E_b$  for helium, Carbon and Iron matter in super strong magnetic fields are shown in **Table 7**.

**Table -5:** Cohesive energies  $-E_b$  (KeV) for atoms in matter in super strong fields with exchange term neglected

<b>B (<math>10^{12}\text{G}</math>)</b>	<b>Hydrogen</b>	<b>Helium</b>	<b>Carbon</b>	<b>Oxygen</b>	<b>Silicon</b>	<b>Iron</b>
1	0.01	0.02	0.16	0.26	0.72	2.18
5	0.01	0.04	0.30	0.50	1.36	4.15
10	0.01	0.06	0.39	0.66	1.79	5.47
50	0.03	0.10	0.74	1.25	3.42	10.41
100	0.04	0.14	0.98	1.65	4.51	13.74
500	0.07	0.26	1.87	3.13	8.58	26.20
600	0.08	0.29	1.92	3.53	9.09	28.40
700	0.082	0.31	2.10	3.69	9.69	30.47
800	0.089	0.32	2.22	3.84	10.32	32.86
900	0.095	0.33	2.34	3.99	10.84	34.11
1000	0.10	0.34	2.46	4.14	11.32	35.40

**Table- 6:** Cohesive energies  $-E_b$  (KeV) for atoms in matter in super strong fields with exchange term included

<b>B (<math>10^{12}\text{G}</math>)</b>	<b>Hydrogen</b>	<b>Helium</b>	<b>Carbon</b>	<b>Oxygen</b>	<b>Silicon</b>	<b>Iron</b>
1	0.04	0.10	0.36	0.51	1.36	2.78
5	0.06	0.16	0.67	1.10	2.38	6.11
10	0.08	0.19	0.87	1.27	3.10	7.78
50	0.12	0.32	1.53	2.31	5.68	14.76
100	0.16	0.40	1.95	2.98	7.28	19.45
500	0.24	0.76	3.37	5.34	12.95	36.10
600	0.25	0.78	3.48	5.67	13.27	38.29
700	0.26	0.79	3.69	5.86	14.34	40.58
800	0.27	0.80	3.95	6.14	15.18	42.17
900	0.29	0.81	4.18	6.39	15.96	43.39
1000	0.30	0.82	4.41	6.74	16.69	45.49

**Table -7:** Comparison of binding energies  $E$ , exchange energy  $E_{\text{ex}}$  and Cohesive energies,  $E_{\text{b}}$  for Helium, Carbon and Iron matter in super strong magnetic field

Matter	B=10 <sup>12</sup> G			B=5×10 <sup>12</sup> G		
	–E (KeV)	–E <sub>ex</sub> (KeV)	–E <sub>b</sub> (KeV)	–E (KeV)	–E <sub>ex</sub> (KeV)	–E <sub>b</sub> (KeV)
<b>Helium</b>						
Our results	0.69	0.13	0.10	1.26	0.20	0.16
Müller	0.60	0.16	0.05	1.11	0.26	0.19
<b>Carbon</b>						
Our results	4.5	0.50	0.36	8.4	0.77	0.62
Hillebrant & Müller	4.1	—	—	7.8	—	—
<b>Iron</b>						
Our results	59.0	3.1	2.8	112.0	5.9	6.1
Flowers	50.2	2.9	2.6	95.7	5.3	8.0
<i>et.al.</i>						

## DISCUSSION OF RESULTS

In this paper, we have evaluated the cohesive energies of Hydrogen, Helium, Carbon, Oxygen, Silicon and Iron matter with and without the exchange energy terms. Our theoretical result indicates the cohesive energy increases with increases of magnetic field. The increase is small for Hydrogen, Helium, but becomes large for Carbon, Oxygen, Silicon and Iron. This trend is repeated in both calculations for with and without exchange term. However, it has been noticed that with the inclusion of exchange term the increase in the cohesive energy is much more pronounced as a function of magnetic field. Our theoretical results are in good agreement with the other workers Romani<sup>7</sup>, Pacynoski<sup>8</sup>, Melezhik<sup>9</sup>, Lai and Qian<sup>10</sup>. We have also calculated the binding energy of Silicon matter and Iron matter with and without the exchange energy term. The calculation also indicates the same trend as was seen in the calculation of Hydrogen, Helium, Carbon and Oxygen matter. Again the binding energy increases with the inclusion of the exchange energy term in the calculation. We have also compared our results of binding energy, exchange energy and cohesive energy of helium. Carbon and Iron matter with those of Müller<sup>3</sup>, Hillebrandt and Müller<sup>4</sup> and Flowers *et.al*<sup>11</sup>.

The condensed matter in super strong magnetic field is assumed to consist of atoms of linear nuclear charges where the corresponding length or interval contains a charge  $Ze$ . The electrons are correspondingly, approximated as a one-dimensional Fermi gas where  $M_0$  electrons fill Landau levels and  $(Z-M_0)$  electrons are quantized in the direction of the field. However, heavier atoms possibly have a spherical core enclosed in a cylinder of valence electrons. Only the valence electrons will contribute to

the binding and the polymeric binding will be reduced compared with the extreme case. Our results therefore become exact only in the limit of infinitely strong magnetic fields. But if we extrapolate the results down to  $12^{12}\text{G}$ , the energy varies with the field approximately as  $B^{2/5}$  for atoms in condensed matter (Abraham and Shapiro,<sup>12</sup> Hujaj and Sunclehear,<sup>13</sup> Arras and Lai,<sup>14, 15</sup> Becken and Schnulchar,<sup>16</sup> Baiko and Yakovlev.<sup>17</sup> There are some recent calculations<sup>18-22</sup> for cohesive property of matter in strong magnetic field and the workers discussed the implication of these results to the recent observations of neutron star surface. Some recent results<sup>23-27</sup> also reveals the same fact.

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