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A theoretical study of quantum critical behavior and evaluation of Spin-lattice relaxation rate, magnetic entropy, pressure dependent Neel temperature and Knight shift of some heavy-electron materials.

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Abstract: Using the theoretical formalism of ***Y-F Yang and D. Pines [Proc Natl Acad Sci USA 109, E3060-E3066 (2012) and Y-F Yang etal [PRL, 103, 199004 (2009*)],** we have theoretically studied the quantum critical behavior of some heavy-electron materials. We have evaluated NMR/NQR spin-lattice relaxation rate as a function of temperature for some fixed values of pressure, magnetic entropies at Neel temperature as a function of pressure, Kondo liquid temperature as a function of pressure, specific heat coefficient and NMR Knight shift as a function of magnetic field and Hall coefficient R_H as a function of temperature for some heavy-electron materials. Our evaluated results show that spin-lattice relaxation rate increases with increase of temperature for all the values of pressure taken. The values are large for low pressure but small for large pressure. Our evaluated values of Kondo liquid temperature as a function of pressure increase with increase of pressure. Our evaluated values of magnetic entropies at Neel temperature as a function of pressure decrease with increase of pressure. Our evaluated results of specific heat coefficient and NMR Knight shift decrease with increase of magnetic field. Our evaluated values of Neel temperature T_N and delocalization temperature T_L as a function of magnetic field show that T_N decrease with magnetic field but T_L increase with magnetic field. Our evaluated results of Neel temperature as a function of pressure indicate that the value increase with pressure. Our evaluated values of Hall Coefficient R_H as a function of temperature show that R_H decrease with increase of temperature. Our all evaluated results are in good agreement with the experimental data and also with other theoretical workers. The entire evaluation in this paper is based on phenomenological two-fluid model. Here, there are two very independent liquids Spin liquid (SL) and Kondo liquid (KL) with two very characteristic temperatures. These liquids play very important role in explain the quantum critical behavior of heavy-electron materials.

Keywords: Quantum critical behavior, Neel temperature T_N, Kondo temperature, delocalization temperature T_L, NMR/NQR spin- lattice relaxation rate, specific heat coefficient, NMR Knight shift, Hall coefficient, Foam factor, anisotropy of hyperfine coupling, Dynamic spin susceptibility, Magnetic susceptibility, Spin liquid, Kondo liquid.

INTRODUCTION:

In an earlier paper[1], , using the theoretical formalism of Y –F Yang and D Pines[2] and Y-F Yang etal[3], we have studied theoretically hybridized spin liquid in heavy-electron materials. We have evaluated the hybridization effectiveness parameter f_0 as a function of magnetic field H (T) for two heavy electron materials $YbRh_2Si_2$ and $CeCoIn_5$. Our theoretically evaluated results show that $f_0(H)$ increases with H for these two heavy electron materials. We have also evaluated pressure dependent hybridization effective parameter f_0 (p) and pressure dependent coherence temperature $T^*(P)$ for these heavy-electron compounds. Our theoretically evaluated results show that f_0 (p) increases as a function of pressure p for $CeCoIn_5$ but decreases as a function of p for $YbRh_2Si_2$. Similarly T^* (p) decreases as a function of p for $YbRh_2Si_2$ but increases as a function of p for $CeCoIn_5$. We have also calculated static spin susceptibility for spin liquid $\chi_{SL}(0,0)$ for different values of hybridization parameter f_0, c-axis spin magnetic susceptibility $\chi_c(a.u)$ for three heavy-electron materials $CeRhIn_5$, $CeCoIn_5$ and URu_2Si_2 all as a function of $\frac{T}{T^*}$. In all these cases, we observed that $\chi_{SL}(0,0)$ decrease as a function of $\frac{T}{T^*}$ for all the values of f_0 taken. Similarly c-axis spin magnetic susceptibility $\chi_c(a.u)$ also decrease as a function of $\frac{T}{T^*}$ for all the above heavy-electron materials taken. We have calculated specific heat coefficient as a function of magnetic field $\gamma_H(J/molK^2)$ and magneto-resistivity coefficient $A(\mu\Omega cm/K^2)$ both as a function of magnetic field H (T). Our theoretically obtained results show that both these parameters decrease with magnetic field H (T). Our theoretically evaluated results are in good agreement with the other theoretical workers[4-6].

In this paper, using the theoretical formalism of Y-H Yang and D. pines[2] and Y-H yang etal.[3], we have theoretically studied quantum critical behavior of some heavy-electron materials. We have theoretically evaluated NMR/NQR spin-lattice relaxation rate as a function of temperature for three fixed values of pressure, magnetic entropies at Neel temperature as a function of pressure, Kondo liquid temperature as a function of pressure, Neel temperature T_N and delocalization temperature T_L as a function of magnetic field H(T), Neel temperature T_N as a function of pressure, specific heat coefficient and NMR Knight shift as a function of magnetic field H(T) and Hall coefficient R_H as a function of temperature for some heavy-electron materials. Our theoretically obtained results of spin-lattice relaxation rate as a function of temperature increase with temperature for all the values of pressure taken. The values are large for low pressure but small for high pressure. We have compared our obtained results with the experimental data[7] and good agreement is observed. Our evaluated results of Kondo liquid temperature as a function of pressure indicate that Kondo liquid temperature increase with increase of pressure. Our evaluated results of magnetic entropies at Neel temperature as a function of pressure decrease with increase of pressure. Our theoretically obtained results are in good agreement with the experimental data[8]. Our theoretically obtained results of specific heat coefficient and NMR Knight shift as a function of magnetic field H(T) decrease with increase of magnetic field. Our evaluated results of Neel temperature T_N and delocalization temperature T_L as a function of magnetic field H(T) show that T_N decrease with magnetic field but T_L increase with magnetic field. Our evaluated results of pressure dependent Neel temperature T_N show that T_N increases with pressure. Our evaluated results of Hall coefficient R_H as a function of temperature T(K) show that R_H decrease with increase of temperature. We compared our evaluated results with experimental data[9] and a good agreement is seen. Our theoretically obtained results are also in good agreement with other theoretical workers[10-12].

MATHEMATICAL FORMULA USED IN THE EVALUATION:

In the evaluation of the above parameters, one takes help of two-fluid description[13-15] of heavy –electron materials whose magnitude is determined from fits to the experimental data. In this description, the heavy electron Kondo liquid emerges below a characteristic temperature T^* a collective hybridization-induced instability of the spin liquid. This describes the lattice of local moments coupled to background conduction electrons. T^* is determined by the effective Ruderman-Kittel-Kasuya-Yoshida (RKKY) interaction between the nearest local moments, the emergent scaling behaviour of the Kondo liquid is characterized by the heavy electron order parameter given by equation (1). In general the static spin susceptibility is given by[16]

$$\chi(T,p) = f(T,p)\chi_{KL}(T,p) + [1 - f(T,p)]\chi_{SL}(T,p) \quad \dots (1)$$

Where the strength of the Kondo liquid component is written as

$$f(T,p) = [f_0(p)\{1 - T/T^*\}] \quad \dots (2)$$

T^* is the coherence temperature at which the Kondo liquid emerges. f_0 measures its effectiveness. The dynamic spin susceptibility of quantum spin liquid is given by

$$\chi_{SL}(q,\omega) = \frac{f_l \chi_0}{1 - zJ_0 f_l \chi_0 - i\omega/\gamma_l} \qquad \dots (3)$$

Where χ_0 is the local susceptibility of an individual f-moments, z is coordination number, f_l is the spectral weight, γ_l is the local relaxation rate and J_q is the RKKY exchange coupling. For simplicity, the magnetic moments of the f-spins and the Boltzmann constants are set to be unity. The hybridization spin liquid will begin to order at Neel temperature and using equation (4), one has

$$zJ_q f_l(T_N)\chi_0(T_N) = 1 \qquad \dots (4)$$

Now

$\chi_0 = C/T$ (5) where C is Curie constant, for $T>T^*$ equation (4) reduces to Curie-Weiss form of the static susceptibility

$$\chi_{SL}(0,0) = C/(T+\theta) \text{ where } \theta = CzJ_{q=0} \qquad \dots (5)$$

One has

$$\frac{T_N}{T^*} = \eta f_l(T_N) \text{ where } \eta = czJ_q/T^* \qquad \dots (6)$$

This parameter reflectes the effects of the crystal lattice and magnetic frustration on T_N. $f_l(T_N)$ plays the role of the collective hybridization in reducing the Neel temperature. Because of the RKKY nature both T^*, $J_q\eta$ is pressure independent. The magnitude of the order moment μ^2 is determined by the local order parameter

$$\frac{\mu^2}{\mu^2_0} = f_l(T_N) \qquad \dots (7)$$

where μ_0 is the local moment strength above T^*. One has

$$\frac{T_N}{T} = \eta\frac{\mu^2}{\mu^2_0} \qquad \dots (8)$$

Because T^* increases with increasing pressure, μ decreases. The Fermi liquid specific heat coefficient is given by

$$\gamma = \frac{R\ln 2}{2T^*}[2 - \ln(1 - f_0^{\frac{-2}{3}})] \qquad \dots (9)$$

Here, R is the gas constant. In mean-field approach, the influence of the crystalline lattice and the magnetic frustration are incorporated in the coordination number z and $\eta = czJ_q / T^*$. In case of 2D square lattice, RKKY coupling is written as[17]

$$J_q = J_n[\cos(q_x) + \cos(q_y)] + 2J_{nm}\cos(q_x)\cos(q_y) \quad \dots (10)$$

Here, J_n and J_{nm} are the nearest and next nearest neighbour couplings respectively

$$\theta = 2cz|J_n + J_{nm}|$$

$$\eta T^* = 2cz|J_n - J_{nm}| \quad \dots (11)$$

In the two-fluid model, one can write the behaviour of Kondo liquid factor T_L as a function of pressure

$$T_L(p) = T^*(p)[1 - f_0(p)^{\frac{-2}{3}}] \quad \dots (12)$$

Below T_L, collective hybridization is completed and one has only itinerant heavy-electron behaviour.

Study of the influence of magnetic field on hybridization effectiveness parameter, quantum criticality and delocalization:

In this case, one can write field induced changes in f_0 as

$$f_0(p,H) = f_0(p)[1 + (\eta_H H)^{\alpha}] \quad \dots (13)$$

Where α is scaling parameter, η_H is assigned to be independent of pressure and do not change across the quantum critical point. The magnetic field effect are considered by the lowest order in H^{α}. In the vicinity of quantum critical point, one may expand f_0 (p) as

$$f_0(p) = 1 + \eta_p(p - p^0{}_c) \quad \dots (14)$$

Here, $p^0{}_c$ is the quantum critical pressure at H=0, η_p is a constant. Now, one has the relation between f_0 and the quantum critical field H_{QC}

$$f_0 = [1 + \eta^{\alpha}{}_H H^{\alpha}{}_{QC}]^{-1} \quad \dots (15)$$

$$\text{And } \frac{T_L(H)}{T^*} = 1 - [\frac{1 + \eta^{\alpha}{}_H H^{\alpha}{}_{QC}}{1 + \eta^{\alpha}{}_H H^{\alpha}}]^{\frac{2}{3}} \quad \dots (16)$$

One has

$$p_c(H) = p^{(0)}{}_c - 1/\eta_p[\frac{\eta^{\alpha}{}_H H^{\alpha}}{1 + \eta^{\alpha}{}_H H^{\alpha}}] \quad \dots (17)$$

For large fields,

$$p^{\infty}{}_{c} = p^{(0)}{}_{c} - \eta^{-1}{}_{p} \qquad \dots (18)$$

At critical field, one has

$$\eta_{p} = \frac{1}{p^{(0)}{}_{c}} \frac{\eta^{\alpha}{}_{H} H^{\alpha}{}_{QC}}{1+\eta^{\alpha}{}_{H} H^{\alpha}{}_{QC}} \qquad \dots (19)$$

$$\frac{p_{c}(H)}{p^{(0)}{}_{c}} = 1 - \frac{1+\eta^{\alpha}{}_{H} H^{\alpha}{}_{QC}}{1+\eta^{\alpha}{}_{H} H^{\alpha}} \left(\frac{H}{H_{QC}}\right)^{\alpha} \qquad \dots (20)$$

Equation (17) and (20) provide a key connection between scaling behaviour, the quantum critical line and the field dependence of the delocalization line at the ambient pressure.

The influence of the magnetic field on the other physical quantities can be studied in the following way. The Neel temperature at which the long range local moment order appears when f_l (p,H)<1 is proportional to the strength of the spin liquid component T_N and its field dependence is given by

$$\frac{T_{N}(p,H)}{T^{0}{}_{N}} = 1 - f(T_{N}, p, H) \qquad \dots. (21)$$

Here, $T^{0}{}_{N}$ is the hypothetical antiferromagnetic ordering temperature of the f-electron lattice in the absence of any hybridization. In two-fluid model, both $T^{0}{}_{N}$ and T^{*} are determined by the local model interaction

$$T^{0}{}_{N} = \eta_{N} T^{*} \qquad \dots (22)$$

η_{N} is constant parameter determined by frustration effects. Now specific heat dependence of magnetic field H in Fermi liquid state and effective mass are given by

$$\gamma_{KL}(H) = \frac{R \ln 2}{2T^{*}} \left[2 + \ln \frac{T^{*}}{T_{L}(H)}\right] \qquad \dots (23)$$

$$\frac{m^{*}}{m_{0}} = \left[\frac{T^{*}}{T_{L}(H)}\right]^{\frac{\alpha}{2}} \qquad \dots (24)$$

$\frac{\alpha}{2}$ is scaling exponent and m_0 is the bare reference electron mass. The total specific heat coefficient is given by

$$\gamma_{QC} = \gamma_0 [\frac{T^*}{T_L(H)}]^{\frac{\alpha}{2}} \quad \ldots (25)$$

Here, γ_0 is independent of magnetic field. Magneto-resistivity in the Fermi Liquid regime is given by

$$\rho(T,H) = A(H)T^2 \quad \ldots (26)$$

Where

$$A(H) = \frac{A_0}{(T^*)^2}[\frac{T^*}{T_L(H)}]^{\alpha} \quad \ldots (27)$$

A_0 is the field independent parameter.

Evaluation of Spin-lattice relaxation rate, magnetic entropy, pressure dependent Neel temperature, Kondo liquid temperature and Knight Shift

Nuclear magnetic/quadrupole resonance (NMR/NQR) experiments on the pressure dependence of the spin-lattice relaxation rate provide a further confirmation of the quantum critical behavior. One assumes that the spin-lattice relaxation rate between T^* and T_0 and is dominated by local moment. This is calculated using Mariya formula[18]

$$\frac{1}{T_1} = \gamma^2 \lim_{\omega \to 0} \Sigma_q F(q)^2 \frac{\operatorname{Im} \chi_l(q,\omega)}{\omega} \quad \ldots (28)$$

Here γ is the gyromagnetic ratio, $F(q)^2$ is the form factor and $\operatorname{Im} \chi_l(q,\omega)$ is the imaginary part of the dynamic susceptibility.

$$F(q)^2 = F^2{}_0[\cos(q_x/2)^2 \cos(q_y/2)^2 + x^2 \sin(q_x/2)^2 \sin(q_y/2)^2] \quad \ldots 29(a)$$

X is the only free parameter which defines the anisotropy of the hyperfine coupling. Spin-lattice relaxation rate also follows the two-fluid behavior given by

$$\frac{1}{T_1} = [1 - f(T)]/T_{ISL} + F(t)/T_{IKL} \quad \ldots 29(b)$$

$$\frac{1}{T_1} = 1/T_{ISL} + (1/T_{IKL} - 1/T_{ISL}) f(T) \quad \ldots 29(c)$$

Here, T_{ISL} and T_{IKL} are the intrinsic contribution of the two components Spin-liquid and Kondo liquid. Information about T_{ISL} and T_{IKL} can be obtained from experimental analysis.

The magnetic entropy is obtained from the formula[19]

$$S(T) = [1 - f(T)]S_{SL}(T) + f(T)S_{KL}(T) \qquad \text{…30(a)}$$

Here S_{SL} is the intrinsic entropy of the local moments and may be approximated as Rln2 for weakly interacting moments (R is gas constant) and S_{KL} is the intrinsic entropy of the heavy electrons. The specific heat coefficient is then given by

$$\frac{C}{T} = \frac{dS(T)}{dT} = [1 - f(T)]\frac{C_{SL}(T)}{T} + f(T)\frac{C_{KL}(T)}{T} + \frac{df(T)}{dT}(S_{KL} - S_{SL}) \qquad \text{…30(b)}$$

Here the third term involves the change in the f-electron spectral weight of the two fluids. If one assumes that the Kondo liquid has a constant Wilson ratio[20], its specific heat coefficient exhibit the same scaling

$$\frac{C_{KL}}{T} = [1 + \ln(\frac{T^*}{T})] \qquad \text{…30(c)}$$

The entropy is given by

$$S_{KL}(T) = R\ln 2\frac{T}{2T^*}[2 + \ln(\frac{T^*}{T})] \qquad \text{…30(d)}$$

The Kondo temperature T_K is defined as

$$T_K = \rho^{-1}e^{-\frac{1}{J\rho}} \qquad \text{…30(e)}$$

$$\rho = \frac{3\gamma}{\pi^2 k^2{}_\beta} \qquad \text{…30(f)}$$

Here, ρ is the density of the background conduction electrons whose value may be estimated from the specific heat coefficient γ of the nonmagnetic host. The Boltzmann constant k_β has been set to unity. J is the Kondo coupling. The NMR Knight shift results from the hyperfine couplings between the probe nuclei and the surrounding polarized electrons and is typically proportional to magnetic susceptibility. The Knight shift and the magnetic susceptibility are written as

$$\chi = f(T)\chi_{KL} + [1 - f(T)]\chi_{SL} \qquad \text{… 31(a)}$$

$$K = K_0 + Af(T)\chi_{KL} + B[1 - f(T)]\chi_{SL} \qquad \text{…31(b)}$$

Here K is the total Knight shift and K_0 is the Knight shift due to local moments. χ_{KL} and χ_{SL} is the intrinsic magnetic susceptibility of the Kondo liquid and hybridized spin liquid respectively. For $T > T^*$ one has $K_0 + B\chi$ and for $T < T^*$, one has

$$K_{anom} = K - K_0 - B\chi = (A - B)\, f(T)\, \chi_{KL} \qquad \text{… 31(c)}$$

The Knight Shift anomaly probes the intrinsic susceptibility or the density of states of the emergent heavy electron Kondo liquid.

The Hall Effect in heavy-electron materials have been ascribed to the skew scattering of the conduction electrons by independent j-moments

$$R_H = R_0 + r_1 \rho \chi \qquad \text{…31(d)}$$

R_0 is the ordinary Hall contribution, $R_s = r_1 \rho \chi$ is the extraordinary or anomalous Hall contribution. Here, ρ is the density of states of the background electrons whose values may be estimated from the specific heat coefficient γ, χ is the magnetic susceptibility. An empirical two-fluid formula is then proposed for the hall coefficient[21]

$$R_H = R_0 + r_1 \rho \chi_l + r_h \chi_h \qquad \text{… 31(e)}$$

Here $r_h \chi_h$ is the contribution from the Kondo liquid. For $T > T^*$, it reduces to the usual Fert-Levy formula[22] while in the limit $r_1 = 0$. One obtains the Kondo liquid scaling $R_H = R_0 + r_h \chi_h$ below T^*.

DISCUSSION OF RESULTS

Using the theoretical formalism of Y-F yang and D. Pines[2] and Y-F Yang etal[3], we have theoretically studied the quantum critical behavior of some heavy-electron materials. We have theoretically evaluated NMR/NQR spin-lattice relaxation rate as a function of temperature for three given values of pressure, Magnetic entropies at Neel temperature as a function of pressure, Kondo liquid temperature as a function of pressure, Neel temperature T_N as delocalization temperature T_L as a function of magnetic field, pressure dependent Neel temperature, specific heat coefficient and NMR Knight shift as a function of magnetic field H(T) and Hall Coefficient as a function of temperature for some heavy-electron materials. In **table T1,** we have shown the evaluated results of NMR/NQR spin-lattice relaxation rate as a function of temperature for heavy electron material $CeRhIn_5$ for three values of pressure P=0Gpa, 0.46Gpa and 1.23 Gpa respectively. Our theoretically obtained results show that spin-lattice relaxation time increase with temperature for all the values of pressure taken. Our values for low pressure is large compared to high pressure. We compared our results with the experimental data[7] and a good agreement is observed. temperature T_x (P) (K) as a function of pressure. Our theoretically obtained results show that Kondo liquid temperature increase with increase of temperature. I

Table T1: An evaluated result of calculated NMR/NQR spin –lattice relaxation rate of $CeRhIn_5$ as a function of temperature for different pressure

T(K)	NMR spin-lattice relaxation rate $T_1^{-1}(10^3 s^{-1})$					
	P=0GPa		P=0.46 GPa		P= 1.23 GPa	
	Cal	Measured	Cal	Measured	Cal	Measured
5.0	1.674	1.559	0.658	0.501	0.226	0.184
8.0	1.708	1.675	0.722	0.586	0.247	0.216
10	1.829	1.732	0.845	0.674	0.289	0.243
12	1.965	1.876	0.969	0.843	0.336	0.307
15	2.058	1.995	1.025	0.986	0.365	0.322
18	2.159	2.106	1.087	1.012	0.408	0.386
20	2.276	2.187	1.132	1.097	0.446	0.409
22	2.334	2.273	1.176	1.135	0.492	0.455
25	2.395	2.432	1.248	1.206	0.537	0.508
28	2.437	2.396	1.296	1.255	0.565	0.527
30	2.498	2.448	1.325	1.297	0.598	0.586
32	2.556	2.505	1.384	1.332	0.622	0.609
34	2.659	2.558	1.439	1.396	0.655	0.635
38	2.722	2.622	1.516	1.455	0.684	0.655
40	2.845	2.765	1.639	1.563	0.703	0.692
45	2.987	2.874	1.706	1.658	0.746	0.708
50	3.045	2.989	1.894	1.749	0.867	0.834

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In **table T2,** we repeated the calculation of spin-lattice relaxation rate for other heavy-electron material but similar results are obtained. In **table T3**, we have shown the evaluated results of Kondo liquid

Table T2: An evaluated result of calculated NMR/NQR spin –lattice relaxation rate T_1^{-1} (s^{-1}) of $CeCoIn_5$ as a function of temperature for different pressure

T(K)	NMR spin-lattice relaxation rate $T_1^{-1}(s^{-1})$					
	P=0GPa		P=0.58 GPa		P= 1.2 GPa	
	Cal	Measured	Cal	Measured	Cal	Measured
5.0	215	242	102	109	43	54
10	226	258	107	115	50	63
15	230	267	112	124	58	74
20	247	274	118	135	66	87
25	258	282	124	139	74	92
30	263	298	135	145	87	105
35	269	310	147	158	96	112
40	273	322	158	163	108	123
45	287	324	166	175	117	138
50	298	339	175	189	125	142
55	310	345	189	199	130	157
60	332	362	208	212	148	169

65	342	367	217	219	159	175
70	355	378	224	227	168	187
75	365	389	233	238	179	192
80	370	398	245	254	188	210
90	392	410	258	267	195	214

Table T3:An evaluated result of Kondo liquid temperature $T_x(P)$ (K) as a function of pressure

P(GPa)	Kondo liquid Temperature $T_x(P)$(K)
0.00	0.00
0.10	0.15
0.20	0.38
0.30	0.49
0.40	0.58
0.50	1.08
0.60	1.12
0.70	1.27
0.80	1.39
0.90	2.07
1.00	2.48
1.10	2.86
1.20	3.08
1.30	3.19
1.40	3.37
1.50	3.88
1.60	4.09

In **table T4**, we have shown the evaluated results of magnetic entropies at Neel temperature as a function of pressure. Our theoretically obtained results show that magnetic entropies S/Rln2 decrease with increase of pressures. We compared our obtained results with the experimental data[8] and a very good agreement is found. In **table T5,** we have shown an evaluated results of spin-lattice relaxation rate as a function of magnetic field H(T) for heavy electron material $YbRh_2si_2$. Our theoretically obtained results show that spin-lattice relaxation rate decrease with increase of magnetic field H(T). In **table T6,** we have shown the theoretically obtained results of specific heat coefficient as a function of magnetic field H(T) for heavy-electron material $YbRh_2Si_2$. Our theoretically obtained results show that specific heat coefficient decrease with increase of magnetic field H(T). In table T7, we have shown the evaluated results of percentage Knight shift K(%) as a function of magnetic field H(T) for heavy electron material $YbRh_2Si_2$. Our theoretically obtained results show that percentage NMR Knight shift K(%) decrease with magnetic field H(T). In **table T8,** we have shown the evaluated result of Neel Temperature T_N and delocalization temperature T_L as a function of magnetic field for heavy-electron material $YbRh_2Si_2$. Our theoretically obtained results show that Neel temperature T_N decrease with magnetic field H(T) but T_L increase with magnetic field H(T). In table T9, we have shown the evaluated results of Neel temperature T_N as a function of pressure p(GPa). Our theoretically obtained results show that T_N temperature increase with increase of pressure. In **table T10,** we have shown the theoretically obtained results of Hall coefficient R_H as a function of temperature T(K) for heavy-electron material URu_2Si_2. Our theoretically obtained results show that R_H decrease with increase of temperature T(K). We compared our theoretically obtained results with the experimental data[9] and fairly good agreement is observed. Our all theoretically obtained results are in good agreement with other theoretical workers[10-12]. There is some recent calculations[23-28] which also reveals the similar behavior.

Table T4: A theoretical result of calculated magnetic entropy at the Neel temperature T_N of heavy-electron material $CeRhIn_5$ as a function of pressure, Results were compared with experimental data[8]

P(GPa)	Magnetic entropy S/Rln2	
	Calculated	Measured
0.00	0.356	-----
0.10	0.347	-----
0.20	0.332	-----
0.30	0.328	-----
0.40	0.305	-----
0.50	0.286	-----
0.60	0.253	-----
0.70	0.229	-----
0.80	0.206	-----
1.0	0.183	0.210
1.5	0.165	0.206
2.0	0.148	0.203
2.20	0.129	0.197
2.40	0.107	0.175
2.60	0.092	0.155
2.80	0.075	0.136
3.00	0.055	0.100
3.50	0.045	0.067

Table T5: An evaluated result of spin-lattice relaxation rate $(T_1T)^{-1/2}$ of heavy electron material $YbRh_2Si_2$ as a function of magnetic field H(T)

H(T)	Spin lattice relaxation rate $(T_1T)^{-1/2}$ $(sK)^{-1/2}$
0.10	5.54
0.20	5.37
0.40	5.22
0.60	4.97
0.80	4.78
1.00	4.52
1.20	4.35

1.40	4.22
1.60	4.10
1.80	3.98
2.00	3.67
2.20	3.42
2.40	3.35
2.60	3.22
2.80	2.58
3.00	2.35
3.50	2.20
4.00	2.12
4.50	2.02
5.00	1.98

Table T6: An evaluated result of specific heat coefficient γ of heavy electron material $YbRh_2Si_2$ as a function of magnetic field H(T)

H(T)	Sp. Heat coefficient γ
0.10	2.168
0.20	2.052
0.40	1.953
0.60	1.827
0.80	1.768
1.00	1.693
1.20	1.585
1.40	1.479
1.60	1.420
1.80	1.386
2.00	1.337
2.20	1.308
2.40	1.265
2.60	1.242
2.80	1.207
3.00	1.188
3.50	1.165
4.00	1.143
4.50	1.089

Table T7: An evaluated result of Knight shift K(%) as a function of magnetic field H(T) of heavy-electron material $YbRh_2Si_2$

H(T)	Knight shift K(%)
0.10	4.232
0.20	4.106
0.40	3.982
0.60	3.759
0.80	3.697
1.00	3.585
1.20	3.496
1.40	3.404
1.60	3.357
1.80	3.309
2.00	3.276
2.20	3.208
2.40	3.183
2.60	3.127
2.80	3.086
3.00	2.893
3.50	2.786
4.00	2.705
4.50	2.678

Table T8: An evaluated result of field dependent Neel Temperature and delocalization temperature T_L for heavy electron material $YbRh_2Si_2$

H(T)	Neel temperature T_N	Delocalization temperature T_L
0.01	0.124	-------
0.02	0.116	------
0.03	0.108	-------
0.04	0.093	-------
0.05	0.087	-------
0.06	0.076	-------
0.07	0.065	------
0.08	0.058	------
0.09	0.042	------
0.10	0.026	0.108
0.12	0.008	0.127
0.14	------	0.146

0.15	------	0.189
0.16	-------	0.267
0.18	------	0.386
0.20	------	0.497
0.30	-------	0.584
0.40	-------	0.696
0.50	---------	0.874

Table T9: An evaluated result of Neel temperature T_N (K) as a function of pressure p(GPa) for heavy-electron material $YbRh_2Si_2$

P(GPa)	Neel temperature T_N (K)
0.00	0.056
0.50	0.087
1.00	0.106
2.00	0.158
2.50	0.227
3.00	0.386
3.50	0.492
4.00	0.556
4.50	0.697
5.00	0.789
5.50	0.872
6.00	0.986
6.50	1.056
7.00	0.867
7.50	0.764
8.00	0.592
10.00	0.467

Table T10: An evaluated result of Hall coefficient R_H ($10^{-3}cm^3C^{-1}$) as a function of temperature T(K) for heavy-electron material URu_2Si_2, results were compared with experimental data[9]

T(K)	Hall coefficient R_H ($10^{-3}cm^3C^{-1}$)	
	Calculated	Experimental data
0	5.623	5.404
10	5.154	5.357
20	4.986	5.102
30	4.697	4.986

40	4.535	4.873
50	4.389	4.794
60	4.427	4.706
70	4.586	4.655
80	4.675	4.602
90	4.874	4.583
100	4.986	4.555
120	5.329	4.502
140	5.406	4.465
150	5.432	4.409
160	5.474	4.322
180	5.566	4.296
200	5.659	4.217

CONCLUSION

From the above theoretical investigations and analysis, we have come across the following conclusions:

1. We have theoretically studied the quantum critical behavior of some heavy-electron materials. We have evaluated NMR/NQR spin-lattice relaxation rate $(T_1)^{-1}(s^{-1})$ as a function of temperature T(K) of two heavy-electron materials $CeRhIn_5$ and $CeCoIn_5$ for three different values of pressure. Our theoretically obtained results show that spin lattice relaxation rate increase with increase of temperature for all the values of pressure taken. The values are large for small pressure and small for large pressure. We have compared our theoretically obtained results with the experimental data and a fairly good agreement is observed.
2. We have also evaluated spin-lattice relaxation rate $(T_1T)^{-1/2}$ as a function of magnetic field H(T) for heavy-electron material $YbRh_2Si_2$. Our theoretical obtained results show that spin-lattice relaxation rate decrease with increase of magnetic field.
3. We have evaluated Kondo liquid temperature $T_x(p)(K)$ as a function of pressure and magnetic entropies S/Rln2 as a function of pressure at Neel temperature. Our theoretically obtained results show that Kondo liquid temperature increase with pressure. The values are small for low pressure and large for high pressure. On the other hand magnetic entropies decrease with increase 0f pressure. We compared our theoretically obtained results with the experimental data and a good agreement is found.
4. We evaluated specific heat coefficient and NMR knight shift K(%) as a function of magnetic field H(T) for heavy-electron material $YbRh_2Si_2$. Our theoretically obtained results show that both specific heat coefficient and NMR knight shift K(%) decrease with increase of magnetic field H(T).

5. We evaluated Neel temperature T_N and delocalization temperature T_L as a function of magnetic field H(T) for heavy electron material $YbRh_2Si_2$. Our theoretically obtained results show that Neel temperature T_N decrease with magnetic field but delocalization temperature T_L increase with increase of magnetic field H(T). Our evaluated results of Neel temperature T_N as a function of pressure p(Gpa) increase with pressure p.

6. We evaluated the Hall coefficient R_H as a function of temperature T(K) for heavy-electron material URu_2Si_2. Our theoretically obtained results show that R_H decrease with increase of temperature T(K). We compared our theoretically obtained results with experimental data and a fairly good agreement is noticed.

7. The entire evaluation of this paper is based on phenomenological two-fluid model. Here, we have two very independent type of liquids called spin liquid (SL) and Kondo liquid (KL) with two very characteristic temperatures. These liquids play a very important role in order to study the quantum critical behavior of heavy-electron materials.

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