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Experimental Research on Optimization of Array Beamforming Based on Least Squares Support Vector Machine

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Abstract: In order to investigate the performances of main lobe width, sidelobe level, array gain and running time for beamformer based on Least Squares Support Vector Machine (LSSVM) algorithm in real acoustic environment, it sets the constraints of support vector regression equations and proposes the beam optimization method based on least squares support vector machine. Then the optimization model of beamformer based on least squares support vector machine is established, the beam optimization model and the concrete implementation process of beamformer based on the least squares support vector machine is discussed, and the experiments in anechoic water tank are carried out. The experimental results show that the method reduces the computational complexity and decreases the memory occupancy and improves the training speed with the performance of the beamformer substantially unchanged. Compared with the traditional support vector regression beamforming, it has characteristics of fast response for real time requirement and provides a new and effective ways for the optimization design of beamformer.

Key words: array, beamforming, least squares support vector machine, optimization

1. INTRODUCTION

With the development of computer and electronic technique, traditional beamforming technique could no longer satisfy the need to explore the oceans. Sonar adaptive beamforming that combines computer technology and signal processing technology improves the performance of sonar signal processing. Consequently, studying new ways of optimizing the underwater array beam has great theoretical and practical value in further improving the performance of sonar system, and promoting the use of this technology in practice.

The array signal processing technique has been widely applied in sonar, radar, communication and many military and civil fields. At present the technique is developing in three directions. The first is high resolution bearing estimation algorithm based on array processing and the typical representative in this field is the multiple signal classification (MUSIC) algorithm and the Eigen subspace algorithm [1]. The second is the design and control of the beam pattern of different kinds of array formations, including array gain, sidelobe level, and main lobe beamwidth, and other parameters. And the typical theories includes the numerical computation method controlled by beam pattern and sidelobe coherence which is influenced by random formation [2], and the convex optimization design method of the beam pattern and semidefinite quadratic programming method[3]. In China, there are also relative studies in the field of beamforming technique of underwater sonar systems, offering some methods with reference value [4]. Yan Shenfeng and some others have used the second-order cone method of optimization in different array formation, which has achieved pretty good effect in controlling the sidelobe level and array gain of beam, and in restraining disturbances [5-7]. On the international stage, the relatively classical book about array signals processing is *Optimum Array Processing* by Harry L. Van Trees [8], which introduces all sorts of beamformer's structure and performance in detail, and also discusses many kinds of high resolution bearing estimation algorithm. The third is the algorithm for adaptive array signal processing, which is how to use the array observed data to design the optimal beamforming weight coefficient according to the system requirement in real time.

Support Vector Machine (SVM) based on statistical learning theory has the advantages like complete theory basis, global convergence, and the disposal of high- dimensional and nonlinear data. Many scholars at home and abroad have applied SVM to the research on the beamforming optimization of antenna array and robustness [9, 10]. Manel et al first adopted the support vector machine in array signal processing, especially in the weighted factor design of beamforming with steering vector existing errors and a variety of unmatched factors, and they introduced a cost function in accordance with SVM，which has remarkable effect on sidelobe suppression and maximum signal to noise ratio[11]. While Angel Navia-Vázquez et al raised approximate kernel function orthogonal method [12] by using Gauss kernel function as primary function with low algorithm complexity compared with other nonlinear processing methods. B. Mohammadzadeh et al evaluated the performance of adaptive beamformer based on SVM under such circumstances of mismatch and match and summarized the advantages of SVM in the robustness of beamformer [13]. Some more frontier work done in this field is as follows. César C. Gaudes and others raised the new way of robust beamforming with sidelobe control based on the support vector machine [14], which has certain advantages by using the ε insensitive loss function as the penalty term compared with other robust beamforming technology. Ignacio Santaman'a and others raised the blind beamforming method of SVM, aiming at the constant modulus property of communication signals, and this method has a good performance in small samples [15].

Currently, the SVM-based method could overcome some shortcomings of traditional machine learning methods and it could solve the practical problems of small sample, non-linearity and high dimensionality, which has been widely applied in pattern recognition, intelligent control, function approximation and other fields [16, 17]. However, the SVM-based method needs a large number of matrix operations in the quadratic optimization process to generate kernel function matrix and requires the calculation of kernel function between any two of all the training samples. When the number of samples is large, the storage of kernel function matrix requires a lot of memory. Meanwhile, the convergence rate of algorithm is slower. And it is difficult to realize the high performance real-time communication system. Therefore, it is important to explore new ways for quick convergence.

2. LEAST SQUARES SUPPORT VECTOR MACHINE

Least Squares Support Vector Machine (LSSVM) is a new method for support vector machine proposed by Suykens which is a kind of traditional improvement of support vector machine [18-20]. It changes the inequality constraints into equality constraints of the traditional support vector machine method, and regards the squared error sum loss function as the empirical loss of training set. Thus the solution of quadratic programming problem is transformed into the solution of linear matrix problem to improve the speed of solving the problem and convergence precision.

Supposing the given sample data is $(x[1], y[1]),\cdots,(x[m], y[m])$, among them $\mathbf{x}[i]\in R^k$ is the input vector, $y[i]\in R$ is the output variables, and $y[i]=\mathrm{f}(\mathbf{x}[i]), i=1,\cdots,m$, $\mathrm{f}(x)$ is the unknown function to be estimated. The samples are mapped from the original space to feature space by nonlinear mapping. For the nonlinear mapping $\varphi: R^k \to H$, φ is called feature mapping and H is feature space. The constructed optimal estimation function $\mathrm{f}(x)$ in high-dimensional feature space is as following:

$$y=\mathrm{f}(x)=\mathbf{w}^T\varphi(\mathbf{x})+b \quad (1)$$

In the above formula $\mathbf{w}$ is the weighted vector in H space and $b\in R$ is the offset. Thus, the nonlinear estimation function is transformed into the linear estimation function of high-dimensional feature space.

Using the structural risk minimization principle, the optimization problem of least squares support vector machine is:

$$\begin{aligned} &\min_{\mathbf{w}} \quad J(\mathbf{w},e)=\frac{1}{2}\|\mathbf{w}\|^2+\frac{1}{2}\eta\sum_{i=1}^{m}e_i^2 \\ &s.t. \quad y[i]=\mathbf{w}^T\varphi(\mathbf{x}[i])+b+\xi_i \end{aligned} \quad (2)$$

In the above formula, $\xi_i\in R, i=1,\cdots,m$ is error relaxation factor variable and $\eta\in R$ is the smaller value usually.

In general, because the weighted vector $\mathbf{w}$ may be infinite-dimensional, it is very difficult to calculate formula (2) directly. Therefore, the optimization problem is transformed into its dual space, and the above optimization problem is solved by using Lagrangian method. Lagrange function is defined as:

$$L(\mathbf{w},b,e,\alpha)=J(\mathbf{w},e)-\sum_{i=1}^{m}\alpha_i(\mathbf{w}^T\varphi(\mathbf{x}[i])+b+\xi_i-y[i]) \tag{3}$$

In the above formula $\alpha_i \in R, i=1,\cdots,m$ is Lagrange multiplier.

According to KKT conditions, the Lagrange function L is taken the partial derivatives with respect to $\mathbf{w}$, e_i, b, α_i respectively to optimize the formula (3), and let its partial derivative be zero. The conditions for the optimum solution are as following:

$$\begin{cases} \dfrac{\partial L}{\partial \mathbf{w}}=0 \rightarrow \mathbf{w}=\sum_{i=1}^{m}\alpha_i\varphi(\mathbf{x}[i]) \\ \dfrac{\partial L}{\partial e_i}=0 \rightarrow \eta e_i=\alpha_i \\ \dfrac{\partial L}{\partial \alpha_i}=0 \rightarrow y[i]=\mathbf{w}^T\varphi(\mathbf{x}[i])+b+e_i \\ \dfrac{\partial L}{\partial b}=0 \rightarrow \sum_{i=1}^{m}\alpha_i=0 \end{cases} \tag{4}$$

Using formula (4) to eliminate $\mathbf{w}$ and ξ_i, the equation of solution for formula (2) is:

$$\begin{bmatrix} 0 & (\mathbf{1}_m)^T \\ \mathbf{1}_m & \mathbf{\Omega}+\dfrac{1}{\eta}\mathbf{I} \end{bmatrix}\begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix}=\begin{bmatrix} 0 \\ \mathbf{Y} \end{bmatrix} \tag{5}$$

In the above formula vector $\mathbf{1}_m=(1,1,\cdots,1)^T$, $\boldsymbol{\alpha}=(\alpha_1,\alpha_2,\cdots,\alpha_m)^T$, $\mathbf{Y}=(y_1,y_2,\cdots,y_m)^T$, $\mathbf{\Omega}$ is the kernel function matrix, $\Omega_{ij}=\mathrm{K}(\mathbf{x}_i,\mathbf{x}_j)=\varphi(\mathbf{x}[i])^T\varphi(\mathbf{x}[j])$. The kernel function $\mathrm{K}(\mathbf{x}_i,\mathbf{x}_j)=\varphi(\mathbf{x}[i])^T\varphi(\mathbf{x}[j])$ is arbitrary symmetric function which meets Mercer theorem.

According to the reference [21], least squares support vector machine optimization problem is transformed into solving linear equations:

$$\begin{bmatrix} 0 & y_1 & \cdots & y_m \\ y_1 & y_1 y_1 \mathrm{K}(x_1, x_1) + 1/\eta & \cdots & y_1 y_m \mathrm{K}(x_1, x_m) \\ \vdots & \vdots & \ddots & \vdots \\ y_m & y_m y_1 \mathrm{K}(x_m, x_1) & \cdots & y_m y_m \mathrm{K}(x_m, x_m) + 1/\eta \end{bmatrix} \times \begin{bmatrix} b \\ \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{6}$$

Using the least squares method to solve the linear equations (6), the obtained value are α and b. After the introduction of kernel function, least squares support vector machine regression estimation function formula (1) becomes:

$$y = \mathrm{f}(x) = \sum_{i=1}^{m} \alpha_i \mathrm{K}(\mathbf{x}, \mathbf{x}[i]) + b \tag{7}$$

In the above formula the kernel function $\mathrm{K}(\mathbf{x}, \mathbf{x}[i]) = \varphi(\mathbf{x})^T \varphi(\mathbf{x}[i])$ is the dot product of arbitrary symmetric kernel function corresponding to the feature space, which is satisfied the Mercer condition.

From the above formula, least squares support vector machine (LSSVM) only needs to determine the shape parameter of kernel function and regularization parameter η without selecting the ε value of insensitive loss function to facilitate the calculation.

3. THE BEAMFORMER OF LEAST SQUARES SUPPORT VECTOR MACHINE

Based on the above advantages, the least squares support vector machine (LSSVM) principle is applied to the optimization process of array signal beamforming. Assuming the received signal is $\mathbf{x}[n] \in C^M$ with the weight $\mathbf{w} \in C^M$, which is defined as following:

$$\begin{cases} \tilde{\mathbf{x}}[n] = \begin{bmatrix} \mathrm{Re}(\mathbf{x}[n]) \\ \mathrm{Im}(\mathbf{x}[n]) \end{bmatrix} \in R^{2M} \\ \hat{\mathbf{x}}[n] = \begin{bmatrix} \mathrm{Im}(\mathbf{x}[n]) \\ -\mathrm{Re}(\mathbf{x}[n]) \end{bmatrix} \in R^{2M} \end{cases} \tag{8}$$

$$\tilde{\mathbf{w}} = \begin{bmatrix} \mathrm{Re}(\mathbf{w}) \\ \mathrm{Im}(\mathbf{w}) \end{bmatrix} \in R^{2M} \tag{9}$$

Then the output of the linear beamformer can be rewritten as:

$$\mathbf{w}^H \mathbf{x}[n] = \tilde{\mathbf{w}}^T \tilde{\mathbf{x}}[n] + j\tilde{\mathbf{w}}^T \hat{\mathbf{x}}[n] \tag{10}$$

The real part and imaginary part are optimized respectively, and then it can be expressed as:

$$\begin{aligned} &\tilde{\mathbf{w}}^T \tilde{\mathbf{x}}[n] \to \mathrm{Re}(\boldsymbol{y}[n]) \\ &\tilde{\mathbf{w}}^T \hat{\mathbf{x}}[n] \to \mathrm{Im}(\boldsymbol{y}[n]) \end{aligned} \tag{11}$$

The desired data is type real $\{+1,-1\}$, namely $\mathrm{Im}(\boldsymbol{y}[n]) \equiv 0$. Therefore, $\tilde{\mathbf{x}}[n]$ is substituted into the optimization problem to solve the weight $\tilde{\mathbf{w}}$.
According to the above assumptions, the optimization problem of linear LSSVM beamforming is:

$$\begin{aligned} &\min_{\tilde{\mathbf{w}}} \qquad J(\tilde{\mathbf{w}}, \xi) = \frac{1}{2} \| \tilde{\mathbf{w}} \|^2 + \frac{1}{2} \eta \sum_{i=1}^{m} \xi_i \\ &s.t. \qquad \boldsymbol{y}[i] = \tilde{\mathbf{w}}^T \, \mathbf{x}[i] + b + \xi_i \end{aligned} \tag{12}$$

The Lagrange multiplier $\boldsymbol{\alpha}$ is substituted into $\mathbf{w} = \sum_{i=1}^{m} \alpha_i \mathbf{x}[i]$ after solving the formula (12). The weight of linear LSSVM beamformer is obtained.
The obtained performance of linear LSSVM beamformer in the low-dimensional space is not ideal. Thus, the data is mapped to a high dimensional feature space H by non-linear mapping φ: $\mathrm{R}^k \to H$. Then linear learning machine is used to train in the high-dimensional feature space. At this time, the processing data is $\varphi(\mathbf{x}[n]) \in C^f$, f is the dimension after the data mapping to high dimensional feature space. According to linear processing method, it can get:

$$\begin{cases} \tilde{\varphi}(\mathbf{x}[n]) = \begin{bmatrix} \mathrm{Re}(\varphi(\mathbf{x}[n])) \\ \mathrm{Im}(\varphi(\mathbf{x}[n])) \end{bmatrix} \in R^{2f} \\ \hat{\varphi}(\mathbf{x}[n]) = \begin{bmatrix} \mathrm{Im}(\varphi(\mathbf{x}[n])) \\ -\mathrm{Re}(\varphi(\mathbf{x}[n])) \end{bmatrix} \in R^{2f} \end{cases} \tag{13}$$

$$\tilde{\mathbf{w}} = \begin{bmatrix} \mathrm{Re}(\mathbf{w}) \\ \mathrm{Im}(\mathbf{w}) \end{bmatrix} \in R^{2f} \tag{14}$$

Then the output of the nonlinear beamformer can be rewritten as:

$$\mathbf{w}^H \varphi(\mathbf{x}[n]) = \tilde{\mathbf{w}}^T \tilde{\varphi}(\mathbf{x}[n]) + j\tilde{\mathbf{w}}^T \hat{\varphi}(\mathbf{x}[n]) \tag{15}$$

Similarly, the real part and imaginary part are optimized respectively, and then it can be expressed as:

$$\tilde{\mathbf{w}}^T \tilde{\varphi}(\mathbf{x}[n]) \to \mathrm{Re}(\boldsymbol{y}[n])$$
$$\tilde{\mathbf{w}}^T \hat{\varphi}(\mathbf{x}[n]) \to \mathrm{Im}(\boldsymbol{y}[n]) \quad (16)$$

The desired data is type real, namely $\mathrm{Im}(\boldsymbol{y}[n]) \equiv 0$. Therefore, $\tilde{\varphi}(\mathbf{x}[n])$ is substituted into the optimization problem to solve the weight $\tilde{\mathbf{w}}$.

According to the above analysis, the optimization problem of nonlinear beamforming based on least squares support vector machine is:

$$\min_{\tilde{\mathbf{w}}} \quad J(\tilde{\mathbf{w}}, \xi) = \frac{1}{2} \| \tilde{\mathbf{w}} \|^2 + \frac{1}{2} \eta \sum_{i=1}^{m} \xi_i$$
$$s.t. \quad \boldsymbol{y}[i] = \tilde{\mathbf{w}}^T \tilde{\varphi}(\mathbf{x}[i]) + b + \xi_i \quad (17)$$

Using Lagrange multiplier method, the weighted vector of nonlinear least squares support vector machine beamformer is:

$$\mathbf{w} = \sum_{i=1}^{m} \alpha_i \tilde{\varphi}(\mathbf{x}[i]) \quad (18)$$

Thus it can be seen that the least squares support vector machine is the solution of linear equations without solving the support vector and solving the quadratic programming problem. The complexity of calculating the matrix is approximately $\frac{1}{3}(m+1)^3 + (m+1)^2$ using the LU decomposition and the complexity of calculating the kernel function matrix is m^2. Therefore, the total complexity is about $\frac{1}{3}(m+1)^3 + (m+1)^2 + m^2 \sim o(m^3)$, compared with the complexity $o(m^{3.5})$ of solving quadratic programming problem by using interior point algorithm in SVM, the computational complexity is decreased and the training speed is faster.

4 . THE WATER TANK EXPERIMENT AND DATA ANALYSIS

4.1 The experimental system and the experimental procedure

In order to investigate the performances of mainlobe width, sidelobe level, array gain and robustness for beamformer based on least squares support vector machine algorithm in real acoustic environment, the water tank experiment was carried out in anechoic tank, which provided experimental data to the hydrophone array beam optimization theory and algorithm based on least squares support vector machine.

The experimental system consists of signal source, acoustic emission transducer, hydrophone, filter amplifier, data acquisition instrument and other components. The system block diagram is as shown in **Fig. 1.** In the experiment the 16-element linear hydrophone array is installed on measuring rotating rod and the distance between the transmitting transducer and receiving array is 6 meters. The transmitting transducer and the receiving array are both hung on the location of 2 meters under water surface in the sound field and the transmitting transducer is directed at the receiving array vertically. Firstly, according to the schematic diagram of water tank experiment, the signal source, acoustic emission transducer, hydrophones, filter amplifier and data acquisition instrument of the experimental system are connected properly after checking the reliability of the cable. Then electrical signal is loaded, the machine is switched on and the researcher starts to debug and observe whether the hydrophone waveform received is normal or not. After debugging, the receiving array is rotated constantly along clockwise from $90°$ to $0°$ and along counterclockwise from $91°$ to $180°$ respectively. The collected data of array within 2 seconds is recorded for each $1°$ angle of rotation on the corresponding position.

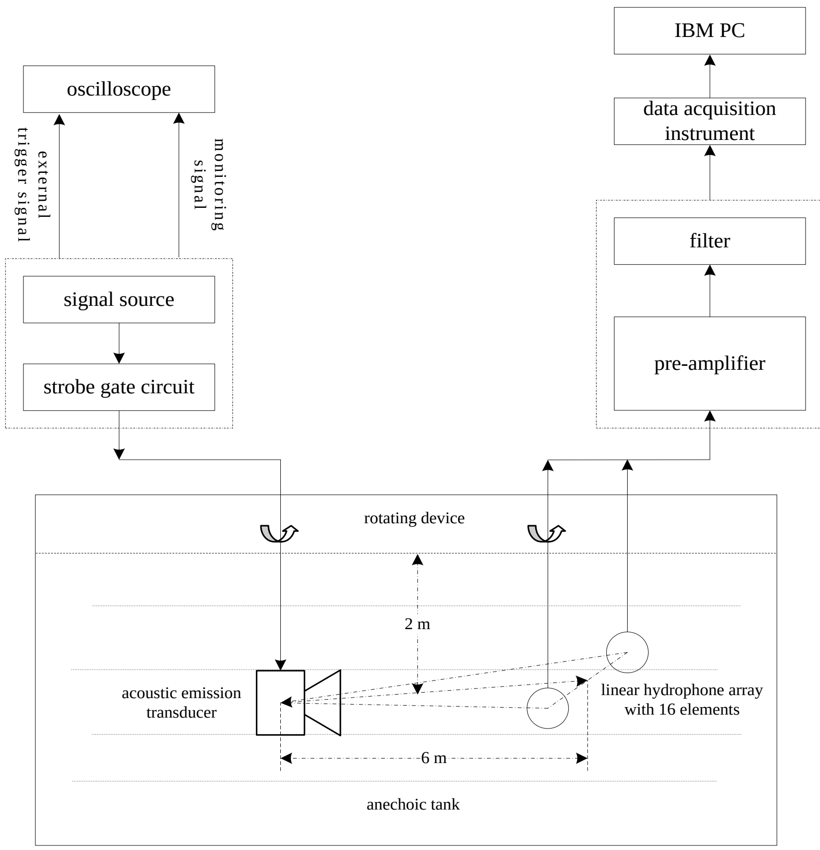

Fig. 1: The system block diagram of water tank experiment

4.2 The experiment data processing and analysis

(1) The processing of experiment data

Considering the uniform linear array of 16 array elements, its spacing of array element is $\lambda/2$ .The single frequency signal is incident on the array form the direction of $\theta = 0°$ vertically with 122880Hz signal sampling frequency and 30KHz center frequency. The original experimental data waveform received by the 8 consecutive array elements from the 4^{th} to the 11^{th} is shown as in **Fig. 2**. The waveform after broadening the original experimental data partially is shown as in **Fig. 3.**

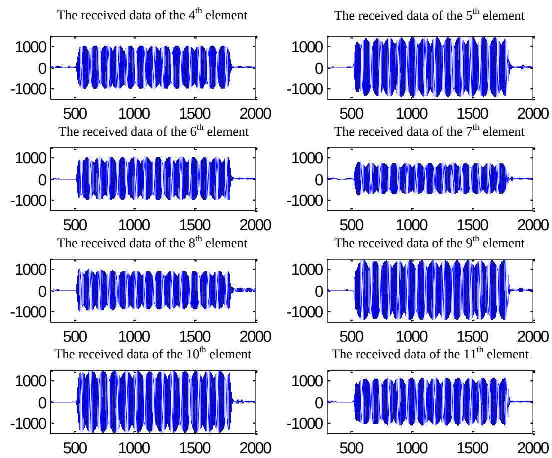

Fig. 2: The original signal waveform of each channel

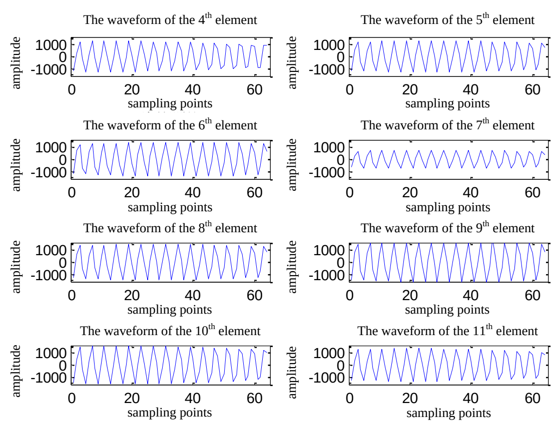

Fig. 3: The signal waveform of each channel data after broadening

(2) The analysis of experiment data

(a) The performance of beamforming: Considering a 8-element uniform linear array, its spacing of array element is $\lambda/2$. The single frequency signal is incident on the array from the directions of $\theta = +10°$ and $\theta = -10°$ respectively with 64 sampling points, 122880Hz sampling frequency and 30KHz signal frequency. The least squares support vector machine adopts the Gaussian radial basis kernel function and the loss function is the linear $\varepsilon -$ insensitive loss function. The Lagrange tradeoff coefficient is $C = 10$ and the parameters of loss function are $\varepsilon = 0.05$, $gam = 10$, $sig2 = 0.2$. The beam patterns of single desired source signal on the observation direction are shown from Fig. 4 to Fig. 7.In the following figures the continuous line denotes Least Squares Support Vector Machine BeamForming with noise (abbreviated as LSSVM-BF), the dotted line denotes Support Vector Regression BeamForming with noise (abbreviated as SVR-BF) and the dashed line means Conventional BeamForming with noise (abbreviated as CBF).

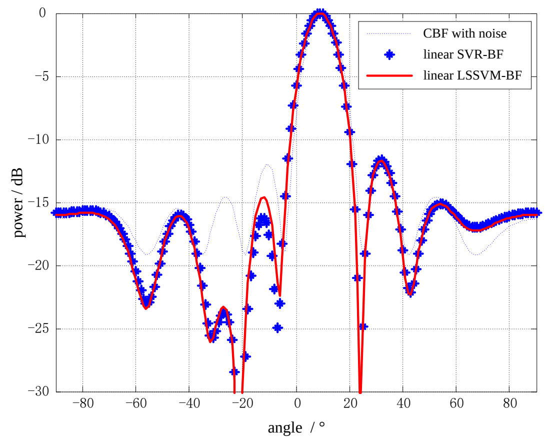

Fig. 4: The linear LSSVM beamforming of $+10°$ direction

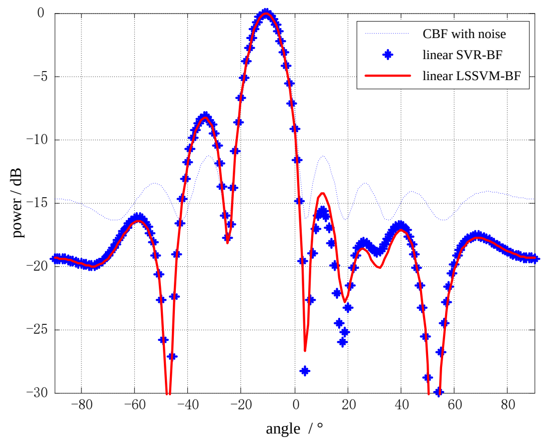

Fig. 5 : The linear LSSVM beamforming of $-10°$ direction

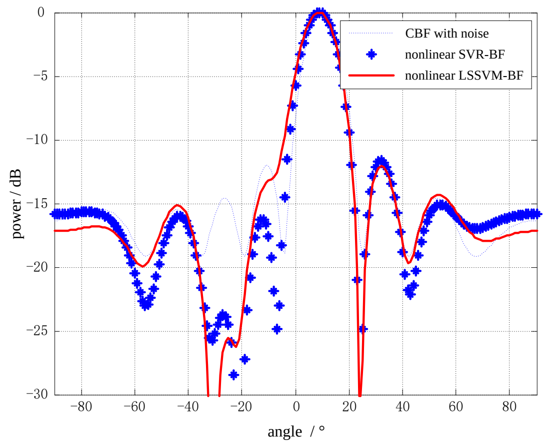

Fig. 6: The nonlinear LSSVM beamforming of $+10°$ direction

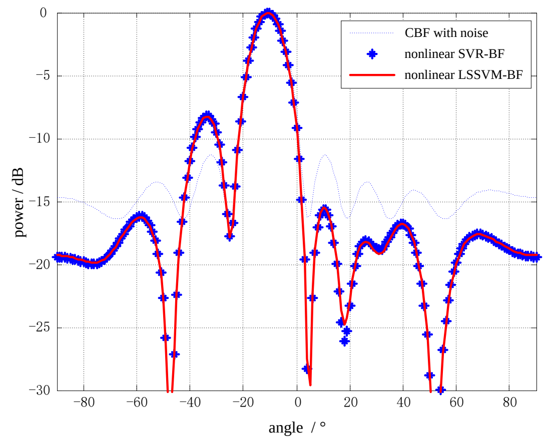

Fig. 7: The nonlinear LSSVM beamforming of $-10°$ direction

From **Fig. 4 to Fig. 7,** it can be seen that, in the water tank environment, whether linear or nonlinear LSSVM beamforming, its properties, such as main lobe width, sidelobe level and array gain, are equivalent to that of noiseless CBF and SVR-BF, and the optimization effects are better.

(b) The running time of beamforming: In order to verify the rapidity of LSSVM, the sampling points of the above array are continuously changed from 16 to 160 by step 16, and the rest parameters are not changed. In the conditions of the same hardware and software, each type of sampling points is simulated 50 times respectively to record the running time of LSSVM and SVR. The average results of the running time are shown in **Tab. 1**. The changing relationship between the running time and the sampling points of LSSVM and SVR beamforming is as shown in **Fig. 8.**

Tab. 1: The average results of the running time of LSSVM and SVR

Sampling points	16	32	48	64	80	96	112	128	144	160
Linear SVR (s)	0.5158	0.6022	0.7468	1.0623	1.5768	2.3075	3.5350	5.3104	7.8956	11.2578
Linear LSSVM (s)	0.0081	0.0083	0.0095	0.0105	0.0114	0.0139	0.0179	0.0225	0.0251	0.0297
Nonlinear SVR (s)	0.4341	0.4971	0.6331	0.9279	1.4579	2.2945	3.9085	5.4403	8.6243	14.7893
Nonlinear LSSVM (s)	0.0020	0.0033	0.0039	0.0049	0.0066	0.0087	0.0115	0.0150	0.0200	0.0237

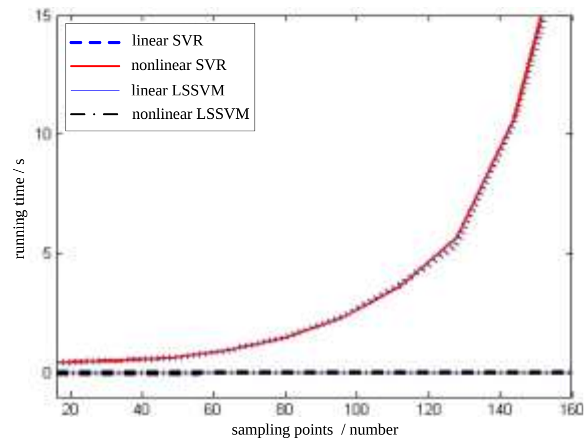

Fig. 8: The changing relationship between the running time and the sampling points of LSSVM and SVR

From **Tab. 1,** the running time of linear LSSVM or nonlinear LSSVM is about 100 times faster than the running time of linear SVR or nonlinear SVR, nearly two orders of magnitude. From **Fig. 8**, when the number of sample points is smaller (about 50), the difference of LSSVM and SVR running time is small; when the number of sample points is larger (about 120), the difference of LSSVM and SVR running time is big. From the trend, with the increase of sampling points, the difference of running time between LSSVM and SVR will be enlarging.

Therefore, the above experiment results show that, whether the performance or the running time, the least squares support vector machine has good rapidity with the unchangeable performance of beamformer. When the amount of data is larger, the advantage of LSSVM is more obvious, which can meet real-time requirements.

5 CONCLUSIONS

In this study, the water tank experiment is carried out to verify the effectiveness of the LSSVM-based hydrophone array beam optimization theory, and the results of water tank experiment are analyzed. Then an investigation of the performance of the LSSVM-based beamformer in actual underwater acoustic environment, such as the mainlobe width, the sidelobe level, the array gain, the robustness and the running time, is done. It has achieved good results. Yet, the hydrophone array beamforming optimization theory and the method based on least squares support vector machine mentioned in the monograph are still at the experimental stage and have not achieved any effective practical application. How to transform the described development potential in theory to the practical engineering application still has a long way to go.

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