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# A theoretical study of electrodynamics of ultra-thin films and evaluation of superconducting properties and optical properties of ultra-thin films at THz frequencies

**Peeyush Ranjan[1] and L. K. Mishra[2]**

[1]S/o Sri Ram Swarap Yadav, Turikala, P.O-Zindapur, Dist-Gaya (Bihar), India

[2]Department of Physics, Magadh University, Bodh Gaya-824234 (Bihar), India

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**Abstract:** Using the theoretical formalism of US Pracht etal [arXiv:1302.6155v2 [cond-mat.supr-con] 3.may 2013, [Phys. Rev B 86, 184503 (2012) and A Semenov et. al [The European Phys. Journal B47, 495 (2005)], we have theoretically studied electrodynamics and evaluated superconducting and optical properties of ultra thin film at THz frequencies. Our theoretically evaluated results of real and imaginary part of complex optical conductivity $\sigma_1$ and $\sigma_2$ as a function of $(2\Delta(T)/\hbar\omega)$ for different temperature and fixed film thickness show that both $\sigma_1$ and $\sigma_2$ decrease with the function $(2\Delta(T)/\hbar\omega)$ for all the temperature taken. Our theoretically evaluated results of superconducting transition temperature $T_C$ (K) increase with film thickness d .Our theoretically obtained results of real and imaginary part of dielectric constant $\varepsilon_1$ and $\varepsilon_2$ as a function of wavelength and fixed film thickness indicate that $\varepsilon_1$ decrease with wavelength and crosses positive and negative value as wavelength increases. However the value of $\varepsilon_2$ increase with wavelength for all film thickness d and its values are always positive.

Our evaluated results of absorbance as a function of wavelength for different film thickness decrease with wavelength Our evaluated results of real and imaginary part of the impedance for a film thickness 5.6nm indicate that real part of the impedance $\Omega$ increase while imaginary part decrease with wavelength. Our evaluated results of temperature dependent upper critical field $B_{C2}$ (T) for different film thickness decrease Our evaluated results of Density of states at the Fermi level $N_0$ as a function of film thickness increase with thickness d. Our evaluated results of energy gap parameter $2\Delta(T)$ mev as a function of temperature for NbN and TaN film decrease with T and becomes zero at $T=T_C$. The results are in accordance with the BCS theory. Our evaluated results of the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ as a function of $T_C$ in the vicinity of $T_C$ show anomalous behavior. The ratio is 4.507 for NbN thin film ($T_C$=12.5K) and 4.082 for TaN thin film ($T_C$=9.7K) respectively.

In this paper, we have evaluated superconducting and optical properties at THz frequencies. These films were grown on Sapphire and evaluated by means of spectral ellipsometry and dc measurement of superconducting critical temperature. The films are well described by the scattering matrix method and with BCS and Drude theory. Our evaluated results are in good agreement with other theoretical workers. These results are quite useful and fruitful in analyzing various devices for nano-photonics.

**Keywords:** Electrodynamics of superconducting states, Ultra-thin film superconductors, THz frequencies, Spectral ellipsometry, dc measurement, Real and imaginary part of complex optical conductivity, Absorbance, Real and imaginary part of Impedance, Temperature dependent upper critical field $B_{C2}$ (T), Density of states at Fermi level, THz time resolved spectroscopy, THz frequency-domain spectroscopy, THz pump probe spectroscopy, Far-IR Fourier spectroscopy.

## INTRODUCTION

Superconductor's thin films are of both technological and academic interest. Here, the interacting electron systems and its properties are governed by electronic density of states (DOS) which changes upon reducing spatial dimensions. The dimensional reduction 3D$\rightarrow$2D may affect optical, electronic and thermodynamic properties of the system. It has been observed that in superconductivity, quasi-2D systems turned out to be very fruitful in the sense that it led to remarkable finding an insulators featuring a superconducting gap[1], pseudo-gap in conventional s-wave superconductors[2] and extremely strong-coupling superconductivity in Kondo lattices[3]. Now days, thin-film superconductors play a key role in many applications such as SQUIDs, thermal and chemical switches using Josephson junctions or microwave resonators[4]. Ultra-thin films of niobium nitride (NbN) and tantalum nitride (TaN) are quite useful in building single-photon detectors[5-8]. Apart from these applications, NbN has been gained attention as a model system for superconductor-insulator transition (SIT), tunneling studies[9] and also in gapped DOS in above $T_C$ in highly disordered films. Similar is the case with TaN films. In this case, it resembles the well-established pseudo-gap in high $T_C$ cuprate superconductors. In this way, the results obtained for NbN films might serve as a key to understand the puzzling pseudo-gap states in high $T_C$ cuprates.

THz time-resolved spectroscopy[10], THz frequency-domain spectroscopy (THz-FDS)[11], THz pump-THz probe spectroscopy[12], far-IR laser spectroscopy[13,14] and far-IR Fourier spectroscopy[15] measurements on thick films of NbN have been performed in comparable spectral range. But measurements for lower frequencies and temperature are lacking which can allow one's to study films with higher degree of disorder with smaller energy gap[9]. It was observed that THz spectroscopy in the range 0.03 to 1.5 THz (1 $cm^{-1}$ to 50$cm^{-1}$) has been proved to be powerful tool to study many questions among the thin films superconducors[16]. This is because the superconducting energy gap $2\Delta$ of many compounds happens to fall in the corresponding energy range of the order of millielectron volt. Importantly, no contacts or surface structuring are required which might affect the sample's properties, or complicate the analysis because of contact effects.

In this paper, using the theoretical formalism of U S Pracht etal[17,18] and A Semenov[19], we have theoretically studied electrodynamics and evaluated superconducting and optical properties of ultra-thin film at THz frequencies. Our theoretically evaluated results of real and imaginary part of complex optical conductivity $\sigma_1$ and $\sigma_2$ as a function of $(2\Delta(T)/\hbar\omega)$ for different temperature and fixed film thickness show that both $\sigma_1$ and $\sigma_2$ decrease with the function $(2\Delta(T)/\hbar\omega)$ for all the temperature taken. Our theoretically evaluated results of superconducting transition temperature $T_C$ (K) increase with film thickness d, 0ur theoretically obtained results of real and imaginary part of dielectric constant $\varepsilon_1$ and $\varepsilon_2$ as a function of wavelength and fixed film thickness indicate that $\varepsilon_1$ decrease with wavelength and crosses positive and negative value as wavelength increases. However the value of $\varepsilon_2$ increase with wavelength for all film thickness d and its values are always positive. Our evaluated results of absorbance as a function of wavelength for different film thickness decrease with wavelength.

Our evaluated results of real and imaginary part of the impedance for a film thickness 5.6nm indicate that real part of the impedance $\Omega$ increase while imaginary part decrease with wavelength. Our evaluated results of temperature dependent upper critical field $B_{C2}$ (T) for different film thickness decrease. Our evaluated results of Density of states at the Fermi level $N_0$ as a function of film thickness increase with thickness d.

Our evaluated results of energy gap parameter $2\Delta(T)$ mev as a function of temperature for NbN and TaN film decrease with T and becomes zero at T=$T_C$. The results are in accordance with the BCS theory. Our evaluated results of the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ as a function of $T_C$ in the vicinity of $T_C$ show anomalous behavior. The ratio is 4.507 for NbN thin film ($T_C$=12.5K) and 4.082 for TaN thin film ($T_C$=9.7K) respectively. Our evaluated results are in good agreement with other theoretical workers[20-22].

**Mathematical formula used in the evaluation:** We have London equation

$$\frac{\partial j_s(\omega,t)}{\partial t} = (n_s e^2 / m)\,\mathrm{E}(\omega,\mathrm{t}) \tag{1}$$

$$\rightarrow j_s(\omega,t) = \frac{n_s e^2}{m}\int_{-\infty}^{t} E(\omega,t')dt' \tag{2}$$

Here, $n_s$ is the super fluid density, e is the elementary charge, m is the electron mass, $j_s(\omega,t)$ is the super current density and $\mathrm{E}(\omega,\mathrm{t})$ is an external perturbation. In the case of electromagnetic wave $E(\omega,t) = E_0 \exp(-i\omega t)$, equation (2) then becomes

$$\mathrm{j}_s(\omega,t) = \frac{i}{\omega}\frac{n_s e^2}{m}E(\omega,t) \tag{3}$$

This equation has a structure of ohm's law. Here, $\frac{i}{\omega}\frac{n_s e^2}{m}$ acts like proportionality factor between $j_s(\omega,t)$ and $E(\omega,t)$.

**Data analysis of single layer systems:** Transmission and phase shift data are fitted by well-known Fresnel equation for multiple reflections at media boundaries. For a single layer system, the transmission T and phase shift $\Phi$ is well written as[23]

$$\mathrm{T} = \frac{[(1-R)^2 + 4R\sin^2\Phi]\exp(-\alpha d)}{[1-\mathrm{R}\exp(-\alpha\,\mathrm{d})^2 + 4\,\mathrm{R}\exp(-\alpha\,\mathrm{d})\sin^2(\beta+\phi)} \tag{4}$$

$$\Phi = \frac{2\pi nd}{\lambda_0} - \arctan[\frac{\kappa(n^2+\kappa^2-1)}{(\kappa^2+n^2)(2+n)n}] + \arctan[\frac{\mathrm{R}\exp(-\alpha\,\mathrm{d})\sin^2(\beta+\phi)}{1-\mathrm{R}\exp(-\alpha\,\mathrm{d})\cos^2(\beta+\phi)}] \tag{5}$$

$$\mathrm{R} = (1-2\,\mathrm{n}+\mathrm{n}^2+\kappa^2)(1+2n+n^2+\kappa^2)^{-1} \tag{6}$$

$$\phi = \arctan[1-2\kappa(1-n^2-\kappa^2)^{-1}] \tag{7}$$

$$\beta = \frac{2\pi nd}{\lambda_0} \qquad 8(a)$$

$$\alpha = \frac{4\pi\kappa}{\lambda_0} \qquad 8(b)$$

$$\sqrt{\varepsilon} = \tilde{n} = n + i\kappa \qquad 8(c)$$

Here R is the reflectivity, $\phi$ is the phase change upon reflection at an interface between the two media. $\beta$ is the angle by which the phase of the radiation is changed upon travelling through a medium of thickness d for a given wavelength $\lambda_0$ and $\alpha$ is the power absorption coefficient. The quantities n and $\kappa$ are the real and imaginary part of the complex refractive index in the given case of non-magnetic material. Equation (4) and (5) do not account for any frequency dependence of optical constant n and $\kappa$, which typically depend on frequency and reveal the properties of interest in the study. To include resonant absorption (e.g Lorentz oscillators) or conductivity (Drude behavior) corresponding terms are used. For example, to describe the free carrier response within the Drude conductivity model, one can introduce complex permittivity $\tilde{\varepsilon} = \varepsilon_1 + i\varepsilon_2$. It has two parts given by

$$\varepsilon_1(\omega) = \varepsilon_\infty - \frac{\omega^2{}_p}{\omega^2 - \tau^2} \tag{9}$$

$$\varepsilon_2(\omega) = \frac{1}{\omega\tau}\frac{\omega^2{}_p}{\omega^2 + \tau^2} \tag{10}$$

This can also be written in terms of optical conductivity $\tilde{\sigma}(\omega)$ which is written as

$$\tilde{\sigma}(\omega) = \frac{i\omega}{4\pi}(\varepsilon_\infty - \tilde{\varepsilon}(\omega)) \tag{11}$$

It has two parts

$$\sigma_1(\omega) = \sigma_{dc}\frac{1}{1+\omega^2\tau^2} \tag{12}$$

$$\sigma_2(\omega) = \sigma_{dc}\frac{\omega\tau}{1+\omega^2\tau^2} \tag{13}$$

$$\sigma_{dc} = \frac{\omega^2{}_p\tau}{4\pi} \tag{14}$$

$\omega_p$ is the plasma frequency, $\sigma_{dc}$ is the dc conductivity, $\varepsilon_\infty$ is the value of the dielectric constant for frequencies much higher than addressed in this study, $1/\tau$ is the relaxation rate.

**Superconducting Properties:** According to GL model[24], the upper critical field is connected with the superconducting coherence length $\xi$ and the magnetic flux quanta $\Phi_0 = h/e$ as

$$B_{C2} = \frac{\Phi_0}{2\pi\xi(\mathrm{T})^2} \tag{15}$$

The exact microscopic relation between the GL coherence length $\xi(T)$ and the BCS coherence length $\xi_0$ which is given by

$$\xi_0 = \hbar v_F[\pi\Delta(0)]^{-1} = e^{\gamma}\hbar v_F[\pi^2 k_\beta T_C]^{-1} \tag{16}$$

Here $\gamma$ =0.577 the Euler's constant. $\Delta(0)$ is the superconducting energy gap at T=0 and $v_F$ is the Fermi velocity. For dirty superconductors the GL coherence length in the vicinity[25] of $T_C$ is given by

$$\xi(\mathrm{T}) = (\frac{\pi^3}{24e^{\gamma}})(\xi_0 l)^{\frac{1}{2}}[\frac{T_C - T}{T_C}]^{\frac{1}{2}} \tag{17}$$

A more realistic value of the upper critical field $B_{C2}$ at zero temperature is given by

$$B_{C2}(0) = 0.69T_C[\frac{dB_{C2}}{dT}]_{T=T_C} \tag{18}$$

The critical current density in GL model is given by

$$j_c(T) = \Phi_0[3^{\frac{3}{2}}\pi\mu_0\Lambda(T)^2\xi(T)]^{-1} \tag{19}$$

Here $\Lambda(T)$ is the effective penetration depth. The expression for $\Lambda(T)$ is given by[26]

$$\Lambda(T)^2 = \frac{2l\zeta(3)\xi_0}{8\pi e^{\gamma}}\frac{1}{\mu_0 e^2 v^2{}_F N_0}[\frac{T_C - T}{T_C}]^{\frac{1}{2}} \tag{20}$$

Here $N_0$ is the electron density of states per one spin at the Fermi level. $\zeta(3)$ is Riemannian zeta function and $\zeta(3)$ =1.202. Now using two-fluid model[27], the temperature dependent of current density is given by

$$j_c(\mathrm{T}) = j_c(0)(1-\mathrm{t}^2)(1-t^4)^{\frac{1}{2}} \tag{21}$$

Here t is the reduced temperature $t = T/T_C$.

$$j_c(0) = 2.26eN_0(k_\beta T_C)^{\frac{3}{2}}(D/\hbar)^{\frac{1}{2}} \tag{22}$$

Here D is the diffusivity which is given by

$$D(cm^2\sec^{-1}) = 1.097/[1 - dB_{C2}(\mathrm{T})/\mathrm{dT}]_{T=T_C} \tag{23}$$

The density of states $N_0$ is determined from the values of diffusivity, transition temperature and the critical current density at zero temperature

We have BCS expression for the rate of $\frac{\Delta(T)}{\Delta(0)}$ is given by

$$\frac{\Delta(T)}{\Delta(0)} = \tanh[\frac{T_C}{T}\frac{\Delta(T)}{\Delta(0)}] \tag{24}$$

The superconducting penetration depth is given by

$$\lambda_L = \sqrt{\frac{c^2}{4\pi\omega\sigma_2}} \tag{25}$$

**Optical properties:** The absorbance of metal films can be written as

$$A = 1 - rr^* - n\tilde{t}\tilde{t}^* \tag{26}$$

Here, r and $\tilde{t}$ are the reflection and transmission coefficient and asterisks are the complex conjugate, n is the real part of the refractive index. Equation (26) can also be written as

$$A = 4Z_0 R_s (Z_0 + (n+1)R_s)^{-2} \tag{27}$$

Here $Z_0 = (\frac{\mu_0}{\varepsilon_0})^{\frac{1}{2}}$ is the impedance of the vacuum. $R_s$ is the surface resistance $R_s = (\sigma d)^{-1}$, $\sigma$ is the conductivity. Impedance of the film is written as

$$Z_F = Z_0 (\frac{\mu_r}{\varepsilon_r})^{\frac{1}{2}} (\Gamma d)^{-1} \tag{28}$$

$\Gamma$ is the propagation constant of the metal

$$\Gamma = -j \frac{2\pi}{\lambda} \sqrt{\mu_r \varepsilon_r} \tag{29}$$

$j = (-1)^{\frac{1}{2}}$ , $\lambda$ is the wavelength of the vacuum. Complex permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$ , susceptibility $\mu = \mu_r \mu_0$

Reflection coefficient $r = \frac{E_r}{E_i}$ and transmission coefficient $\tilde{t} = \frac{E_t}{E_i}$ (30)

Here $E_i$, $E_r$ and $E_t$ are the amplitude of the electric field of the incident wave, reflected wave and the wave propagating in the dielectric.

## DISCUSSION OF RESULTS

Using the theoretical formalism of U S Pracht etal[17,18] and A Semenov et. al[19]. We have theoretically studied the electrodynamics of superconducting state and evaluated the superconducting and optical properties of ultra-thin film at THz frequencies. In **Table T1,** we have evaluated the real part of complex conductivity $\sigma_1$ as a function of $[2\Delta(T)/\hbar\omega]$ for different values of temperature. Here, $2\Delta(T)$ is temperature dependent energy gap parameter. The temperature are T=0.1T$_C$, 0.4T$_C$, o.6T$_C$

and $0.8T_C$ respectively. We have kept the values of $\frac{1}{\tau}$ ($\tau$ is relaxation rate) as 40, 20, 4 and 2x $\Delta(0)$ in this evaluation. Our theoretically evaluated results show that $\sigma_1$ decrease as a function of $[2\Delta(T)/\hbar\omega]$ for all the values of temperature taken. In **Table T2,** we have repeated the calculation for imaginary part of complex conductivity $\sigma_2$ as a function of $[2\Delta(T)/\hbar\omega]$ for the same set of temperatures. The others parameter is the same as in **Table T1**.

Our theoretically obtained values indicate that $\sigma_2$ also decrease with $[2\Delta(T)/\hbar\omega]$ for the same set of temperatures. In **Table T3 and T4,** we have shown the optical constants and superconducting properties of NbN thin film for different values of thickness d(nm). These are transition temperature $T_C$(K), Diffusivity D($cm^2sec^{-1}$) relaxation rate $\tau$ (fm), electron mean free path l (lm), Square resistance at room temperature $R_s$ ($\Omega$) (at 295K), Plasma frequency $\omega_p$ ($10^{15}$ rad $sec^{-1}$), Critical current density $j_C(0)$ ($MAcm^2$) and density of states $N_0$ ($10^{47}J^{-1}m^{-3}$) respectively. The values of these parameters are used in different calculation.

**Table T1:** An evaluated results of Re $(\sigma_{dc}) = \sigma_1$ as a function of frequency $(2\Delta(T)/\hbar\omega)$ with different values of temperature having $\frac{1}{\tau}$=40, 20, 4, 2x $2\Delta(0)$. The values of these parameters are $2\Delta(0)$ =25cm$^{-1}$, $T_C$= 10K and $\sigma_{dc}$ =1066 $\Omega^{-1}cm^{-1}$

| **Frequency** $(2\Delta(T)/\hbar\omega)$ | ← **Re** $(\sigma_{dc}) = \sigma_1$ → | | | |
|---|---|---|---|---|
| | **T=0.1T$_C$** | **T=0.4T$_C$** | **T=0.6T$_C$** | **T=0.8T$_C$** |
| 0 | 0.25 | 0.42 | 0.52 | 0.57 |
| 0.5 | 0.20 | 0.35 | 0.48 | 0.53 |
| 1.0 | 0.15 | 0.31 | 0.42 | 0.49 |
| 1.5 | 0.12 | 0.29 | 0.39 | 0.43 |
| 2.0 | 0.08 | 0.26 | 0.33 | 0.39 |
| 2.2 | 0.14 | 0.38 | 0.30 | 0.47 |
| 2.4 | 0.19 | 0.42 | 0.37 | 0.58 |
| 2.6 | 0.25 | 0.47 | 0.48 | 0.65 |
| 2.8 | 0.27 | 0.49 | 0.57 | 0.69 |
| 3.0 | 0.32 | 0.52 | 0.63 | 0.75 |
| 3.2 | 0.35 | 0.55 | 0.69 | 0.79 |
| 3.4 | 0.39 | 0.58 | 0.78 | 0.84 |
| 3.5 | 0.48 | 0.63 | 0.82 | 0.87 |

**Table T2:** An evaluated results of Im $(\sigma_{dc}) = \sigma_1$ as a function of frequency $(2\Delta(T)/\hbar\omega)$ with different values of temperature having $\frac{1}{\tau}$=40, 20, 4, 2x $2\Delta(0)$. The values of these parameters are $2\Delta(0)$ =25cm$^{-1}$, $T_C$= 10K and $\sigma_{dc}$ =1066 $\Omega^{-1}cm^{-1}$

| **Frequency** $(2\Delta(T)/\hbar\omega)$ | ← **Im** $(\sigma_{dc}) = \sigma_2$ → | | | |
|---|---|---|---|---|
| | **T=0.1T$_C$** | **T=0.4T$_C$** | **T=0.6T$_C$** | **T=0.8T$_C$** |
| 0 | 12.6 | 10.6 | 8.8 | 7.8 |
| 0.5 | 10.8 | 8.3 | 7.5 | 7.2 |
| 1.0 | 8.4 | 7.6 | 6.4 | 6.4 |
| 1.5 | 6.2 | 6.5 | 6.0 | 5.7 |
| 2.0 | 5.6 | 5.2 | 5.4 | 5.2 |
| 2.2 | 4.7 | 4.6 | 4.3 | 4.6 |
| 2.4 | 4.0 | 3.9 | 3.7 | 4.3 |
| 2.6 | 3.5 | 3.4 | 3.2 | 3.8 |
| 2.8 | 3.2 | 3.0 | 2.9 | 3.2 |
| 3.0 | 2.9 | 2.8 | 2.6 | 2.8 |
| 3.2 | 2.7 | 2.6 | 2.2 | 2.5 |
| 3.4 | 2.4 | 2.2 | 1.8 | 2.0 |
| 3.5 | 1.6 | 1.8 | 1.5 | 1.8 |

**Table T3:** Optical constants of ultra-thin film of NbN, thickness d(nm) $T_C$(K), Diffusivity D(cm$^2$cm$^{-1}$), relaxation rate $\tau$ (fsec), electron mean free path l (nm)

| **d (nm)** | **T$_C$(K)** | **D(cm$^2$cm$^{-1}$)** | **$\tau$ (fsec)** | **l (nm)** |
|---|---|---|---|---|
| 3.2 | 9.87 | 0.510 | 2.16 | 0.58 |
| 5.8 | 13.50 | 0.582 | 3.54 | 0.79 |
| 8.3 | 14.40 | 0.599 | 3.53 | 0.80 |
| 11.7 | 15.20 | 0.602 | 3.48 | 0.79 |
| 14.4 | 15.75 | 0.598 | 3.86 | 0.82 |

**Table T4:** Optical constants of ultra-thin film of NbN, thickness d (nm), Square resistance at room temperature $R_S$ $\Omega$ (295K), Plasma frequency $\omega_p(10^{15} rad\sec^{-1})$, Critical current at T=0 $j_c(0)(MAcm^{-2})$, Density of states $N_0(10^{47} J^{-1}m^{-3})$

| **d(nm)** | **R$_s$ $\Omega$** | $\omega_p(10^{15} rad\sec^{-1})$ | $j_c(0)(MAcm^{-2})$ | $N_0(10^{47} J^{-1}m^{-3})$ |
|---|---|---|---|---|
| 3.2 | 707 | 13.46 | 2.92 | 0.73 |
| 5.8 | 265 | 11.45 | 9.51 | 1.39 |
| 8.3 | 165 | 12.96 | 11.35 | 1.48 |
| 11.7 | 105 | 13.55 | 13.46 | 1.63 |
| 14.4 | 84 | 14.66 | 13.30 | 1.60 |

In **Table T5,** we have shown the evaluated result of superconducting transition temperature $T_C$ (K) as a function of film thickness d (nm) for NbN film grown on Sapphire. Our evaluated results indicate that $T_C$ (K) increase with film thickness d (nm). In **Table T6,** we have shown the evaluated results of real part of the dielectric constant $\varepsilon_1$ as a function of wavelength (nm) for different values of NbN film thickness d(nm). Our evaluated results show that for small wavelength $\varepsilon_1$ is positive and as wavelength increases, $\varepsilon_1$ crosses from positive to negative values. The negative values increase for large wavelength.

In **Table T7,** we repeated the calculation for imaginary part of the dielectric constant $\varepsilon_2$ as a function of wavelength (nm) for different values of NbN film thickness d(nm). Our evaluated results show that $\varepsilon_2$ increase with wavelength for all the film thickness d and the values are all positive. In **TableT8,** we have shown the evaluated results of absorbance of NbN thin film as a function of wavelength (nm) for different values of film thickness d(nm). Our theoretically obtained results show that absorbance decrease with wavelength for all thickness d. In **Table T9**, we have shown the evaluated result of real part and imaginary part of the impedance $\Omega$ for 5.6 nm thin film as a function of wavelength (nm). Our evaluated results show that real part of the impedance $\Omega$ increase and decrease with wavelength whereas imaginary part of the impedance $\Omega$ decrease with wavelength and crosses positive to negative values for large wavelength.

**TableT5:** An evaluated results of superconducting transition temperature $T_C$ as a function of film thickness d(nm) for NbN film grown on Sapphire**.**

| **Thickness d(nm)** | **Transition temperature $T_C$ (K)** |
|---|---|
| 2 | 10.2 |
| 3 | 11.4 |
| 4 | 11.8 |
| 5 | 12.4 |
| 6 | 12.9 |
| 7 | 13.3 |
| 8 | 13.8 |
| 9 | 14.2 |
| 10 | 14.6 |
| 11 | 14.9 |
| 12 | 15.4 |
| 13 | 15.8 |
| 14 | 16.3 |
| 15 | 16.7 |
| 16 | 17.5 |

**Table T6:** An evaluated result of Real part of dielectric constant $\varepsilon_1$ as a function of wavelength for different thickness of NbN ultra thin film

| **Wavelength ( nm)** | $\varepsilon_1$ | | | |
|---|---|---|---|---|
| | **d=3.2nm** | **d=5.6nm** | **d=11.7nm** | **d=14.4nm** |
| 200 | 0.125 | 0.186 | 0.192 | 0.225 |
| 400 | 0.087 | 0.032 | 0.027 | 0.016 |
| 600 | -2.485 | -3.34 | -4.25 | -5.17 |
| 800 | -10.87 | -11.29 | -12.22 | -13.29 |
| 1000 | -16.38 | -17.88 | -18.43 | -19.14 |
| 1200 | -20.23 | -22.48 | -23.27 | -24.20 |
| 1500 | -24.87 | -25.26 | -26.14 | -27.58 |
| 1600 | -26.49 | -27.56 | -28.34 | -29.12 |
| 1800 | -32.95 | -33.20 | -34.52 | -35.06 |
| 2000 | -35.86 | -36.15 | -37.15 | -38.49 |
| 2200 | -38.29 | -39.59 | -40.20 | -42.05 |
| 2400 | -42.86 | -43.55 | -44.27 | -45.68 |
| 2500 | -45.47 | -46.28 | -47.85 | -48.16 |

**TableT7:** An evaluated result of imaginary part of dielectric constant $\varepsilon_2$ as a function of wavelength (nm) for NbN film with different thickness d

| **Wavelength ( nm)** | $\varepsilon_2$ | | | |
|---|---|---|---|---|
| | **d=3.2nm** | **d=5.6nm** | **d=11.7nm** | **d=14.4nm** |
| 200 | 1.862 | 2,557 | 3.387 | 4.227 |
| 400 | 8.942 | 9.244 | 10.596 | 12.056 |
| 600 | 12.326 | 14.765 | 15.227 | 16.652 |
| 800 | 22.158 | 24.968 | 25.078 | 26.145 |
| 1000 | 25.927 | 27.554 | 28.142 | 29.972 |
| 1200 | 32.589 | 34.256 | 35.572 | 36.039 |
| 1500 | 43.288 | 45.178 | 46.279 | 47.148 |
| 1600 | 55.147 | 57.059 | 58.842 | 59.532 |
| 1800 | 67.258 | 69.125 | 70.576 | 72.148 |
| 2000 | 70.582 | 73.056 | 74.127 | 75.652 |
| 2200 | 84.963 | 87.234 | 88.965 | 89.097 |
| 2400 | 92.587 | 94.086 | 95.156 | 96.862 |
| 2500 | 105.436 | 107.872 | 108.249 | 110.567 |

**Table T8:** An evaluated result of absorbance as a function of wavelength (nm) for different thickness of NbN film

| Wavelength (nm) | < ------ Absorbance---------→ | | | |
|---|---|---|---|---|
| | **d=3.2nm** | **d=8.05nm** | **d=11.7nm** | **d=14.4nm** |
| 200 | 0.327 | 0.432 | 0.508 | 0.524 |
| 400 | 0.308 | 0.416 | 0.476 | 0.516 |
| 600 | 0.276 | 0.408 | 0.425 | 0.502 |
| 800 | 0.187 | 0.386 | 0.408 | 0.497 |
| 1000 | 0.248 | 0.415 | 0.396 | 0.486 |
| 1200 | 0.297 | 0.428 | 0.418 | 0.515 |
| 1400 | 0.328 | 0.477 | 0.433 | 0.536 |
| 1600 | 0.347 | 0.506 | 0.457 | 0.578 |
| 1800 | 0.355 | 0.515 | 0.486 | 0.595 |
| 2000 | 0.366 | 0.527 | 0.525 | 0.616 |
| 2200 | 0.375 | 0.536 | 0.556 | 0.623 |
| 2400 | 0.382 | 0.542 | 0.578 | 0.637 |
| 2500 | 0.395 | 0.557 | 0.595 | 0.645 |

**TableT9:** An evaluated result of Real part of Impedance $\Omega$ and imaginary part of Impedance $\Omega$ for 5.6 thick NbN film as a function of wavelength (nm)

| Wavelength (nm) | Re Part of Impedance $\Omega$ | Im Part of Impedance $\Omega$ |
|---|---|---|
| 0 | 300.9 | 0.278 |
| 0.5 | 310.4 | 1.487 |
| 1.0 | 332.7 | 2.276 |
| 1.5 | 412.8 | 0.148 |
| 2.0 | 403.2 | -0.265 |
| 2.5 | 397.4 | -2.556 |
| 3.0 | 378.5 | -5.862 |
| 3.5 | 367.9 | -10.434 |
| 4.0 | 358.6 | -27.347 |
| 4.5 | 347.4 | -30.543 |
| 4.8 | 340.8 | -42.558 |
| 5.0 | 336.3 | -57.682 |
| 5.5 | 330.2 | -68.246 |
| 6.0 | 328.5 | -79.205 |
| 6.5 | 325.7 | -60.268 |

In **Table T10**, we have shown the evaluated results of temperature dependence of upper critical field $B_{C2}(T)$ for different values of film thickness of NbN film. Our evaluated results show that $B_{C2}(T)$ decrease with T for all thickness d. The value is large for d=3.2nm and small for d=15nm but trend is the same for all d taken. In **Table T11,** we have shown the evaluated results of Density of states $N_0$ ($10^{47} J^{-1} m^{-3}$) as a function of film thickness d (nm). Our evaluated results indicate that $N_0$ increase with thickness d. In **Table T12,** we have shown the evaluated results of energy gap parameter $2\Delta(T)$ as a function of temperature T (K) for NbN ultra thin film for T< $T_C$(K). $T_C$ (K) for NbN thin film is 12.2K. Our evaluated results show that $2\Delta(T)$ (mev) decrease with T for T<$T_C$ (K).

In **Table T13,** we have shown the evaluated results of the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ as a function of $T_C$ (K) in the vicinity of $T_C$ (K) for NbN thin film. Our evaluated results show that the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ increase with $T_C$ and its value at $T_C$(K)=12.2K is 4.407.

**Table T10:** An evaluated result of temperature dependence of upper critical field $B_{C2}$ (T) for different thickness d(nm) of NbN ultra thin films

| T(K) | $B_{C2}$ (T) | | | |
|---|---|---|---|---|
| | d=15nm | d=4.8nm | d=3.9nm | d=3.2nm |
| 5 | 10.654 | 12.487 | 14.359 | 15.878 |
| 7 | 9.786 | 11.088 | 13.268 | 14.792 |
| 8 | 8.323 | 10.453 | 12.567 | 13.534 |
| 9 | 7.487 | 9.875 | 11.955 | 12.687 |
| 10 | 6.342 | 8.557 | 10.874 | 11.329 |
| 11 | 5.435 | 7.486 | 9.559 | 10.867 |
| 12 | 5.087 | 6.148 | 8.697 | 9.532 |
| 13 | 4.925 | 5.872 | 8.256 | 8.956 |
| 14 | 4.167 | 5.236 | 7.987 | 8.125 |
| 15 | 3.864 | 4.989 | 7.132 | 7.998 |
| 16 | 3.259 | 4.165 | 6.965 | 7.304 |
| 17 | 2.674 | 3.876 | 6.327 | 6.987 |
| 18 | 2.086 | 2.873 | 5.999 | 6.253 |
| 19 | 1.573 | 3.052 | 5.535 | 5.878 |
| 20 | 0.967 | 2.574 | 4.865 | 5.147 |

**Table T11:** An evaluated result of Density of states $N_0$ ($10^{47}J^{-1}m^{-3}$) as a function of thickness d (nm) for NbN ultra-thin films

| **d (nm)** | **Density of States $N_0$ ($10^{47}J^{-1}m^{-3}$)** |
|---|---|
| 2 | 0.689 |
| 3 | 0.734 |
| 4 | 0.728 |
| 5 | 0.298 |
| 6 | 0.876 |
| 7 | 0.975 |
| 8 | 1.057 |
| 9 | 1.125 |
| 10 | 1.167 |
| 11 | 1.232 |
| 12 | 1.257 |
| 13 | 1.296 |
| 14 | 1.309 |
| 15 | 1.356 |
| 16 | 1.397 |
| 17 | 1.574 |
| 18 | 1.747 |

**TableT12:** An evaluated result of energy gap parameter $2\Delta$ (T) mev as a function of temperature T(K) for NbN thin film ($T_C$=12.2K)

| **T(K)** | **$2\Delta$ (T) mev** |
|---|---|
| 0 | 3.827 |
| 1 | 3.804 |
| 2 | 3.765 |
| 3 | 3.727 |
| 4 | 3.675 |
| 5 | 3.542 |
| 6 | 3.038 |
| 7 | 2.842 |
| 8 | 2.546 |
| 9 | 2.157 |
| 10 | 1.875 |
| 11 | 1.586 |
| 12 | 1.042 |
| 13 | 0.007 |

**Table T13:** An evaluated result of the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ as a function of temperature $T_C$ (K) in the vicinity of critical temperature $T_C$ for NbN thin film

| $T_C$ (K) | $\frac{2\Delta(0)}{K_\beta T_C}$ |
|---|---|
| 8.0 | 3.864 |
| 8.5 | 3.954 |
| 9.0 | 4.047 |
| 9.5 | 4.178 |
| 10.0 | 4.255 |
| 10.5 | 4.364 |
| 11.0 | 4.422 |
| 11.5 | 4.458 |
| 12.0 | 4.472 |
| 12.5 | 4.507 |

In **Table T14,** we have shown the evaluated results of energy gap parameter $2\Delta(T)$ (mev) as a function of T(K) for TaN thin film whose $T_C$(K) is 9.7K. Our evaluated results show that the energy gap parameter $2\Delta(T)$ (mev) decrease with T(K).

**Table T14:** An evaluated result of energy gap parameter $2\Delta$ (T) mev as a function of temperature T(K) for TaN ultra thin film ($T_C$ =9.7K)

| **T(K)** | **$2\Delta$ (T) mev** |
|---|---|
| 0 | 2.864 |
| 1 | 2.832 |
| 1.5 | 2.806 |
| 2.0 | 2.687 |
| 2.5 | 2.422 |
| 3.0 | 2.235 |
| 3.5 | 2.058 |
| 4.0 | 1.867 |
| 4.5 | 1.589 |
| 5.0 | 1.322 |
| 6.0 | 1.257 |
| 7.0 | 1.184 |
| 8.0 | 1.126 |
| 9.0 | 0.786 |
| 10.0 | 0.055 |

In **Table T15,** we have shown the evaluated results of the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ as a function of $T_C$(K) in the vicinity of $T_C$ for TaN thin film. Our evaluated results show that the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ increase with $T_C$ in the vicinity of $T_C$ and its value at $T_C$=9.7K is 4.075. Our evaluated results are in good agreement with other theoretical workers[20-22]. There is some recent calculations[28-37] which reveals the similar type of behavior.

**TableT15: An evaluated result of the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ as a function of $T_C$ (K) in the vicinity of critical temperature $T_C$ (K) for TaN thin film**

| $T_C$(K) | $\frac{2\Delta(0)}{K_\beta T_C}$ |
|---|---|
| 7.0 | 3.626 |
| 7.5 | 3.648 |
| 8.0 | 3.786 |
| 8.5 | 3.845 |
| 9.0 | 3.967 |
| 9.2 | 4.008 |
| 9.4 | 4.012 |
| 9.5 | 4.028 |
| 9.6 | 4.046 |
| 9.7 | 4.075 |
| 9.8 | 4.082 |
| 9.9 | 4.106 |
| 10.0 | 4.122 |

## CONCLUSION

From the above theoretical investigation and numerical analysis, we have come to the following conclusions:

1. We have theoretically evaluated superconducting and optical properties of ultra-thin films at THz frequencies. Our theoretically evaluated results of real and imaginary part of complex optical conductivity $\sigma_1$ and $\sigma_2$ as a function of $(2\Delta(T)/\hbar\omega)$ for different temperature and fixed film thickness show that both $\sigma_1$ and $\sigma_2$ decrease with the function $(2\Delta(T)/\hbar\omega)$ for all the temperature taken.
2. Our theoretically evaluated results of superconducting transition temperature $T_C$ (K) increase with film thickness d.

3. Our theoretically obtained results of real and imaginary part of dielectric constant $\varepsilon_1$ and $\varepsilon_2$ as a function of wavelength and fixed film thickness indicate that $\varepsilon_1$ decrease with wavelength and crosses positive and negative value as wavelength increases. However the value of $\varepsilon_2$ increase with wavelength for all film thickness d and its values are always positive.
4. Our evaluated results of absorbance as a function of wavelength for different film thickness decrease with wavelength.
5. Our evaluated results of real and imaginary part of the impedance for a film thickness 5.6nm indicate that real part of the impedance $\Omega$ increases while imaginary part decreases with wavelength.
6. Our evaluated results of temperature dependent upper critical field $B_{C2}$ (T) for different film thickness decrease.
7. Our evaluated results of Density of states at the Fermi level $N_0$ as a function of film thickness increase with thickness d.
8. Our evaluated results of energy gap parameter $2\Delta(T)$ mev as a function of temperature for NbN and TaN film decrease with T and becomes zero at $T=T_C$. The results are in accordance with the BCS theory.
9. Our evaluated results of the ratio $\frac{2\Delta(0)}{K_\beta T_C}$ as a function of $T_C$ in the vicinity of $T_C$ show anomalous behavior. The ratio is 4.507 for NbN thin film ($T_C$=12.5K) and 4.082 for TaN thin film ($T_C$=9.7K) respectively.
10. In this paper, we have evaluated superconducting and optical properties at THz frequencies. These films were grown on Sapphire and evaluated by means of spectral ellipsometry and dc measurement of superconducting critical temperature. The films are well described by the scattering matrix method and with BCS and Drude theory. Our evaluated results are in good agreement with other theoretical workers. These results are quite useful and fruitful in analyzing various devices for nano-photonics.

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*** Corresponding author: Peeyush Ranjan**

S/o Sri Ram Swarap Yadav, Turikala, P.O-Zindapur,

Dist-Gaya (Bihar)



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